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Journal of the

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

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Papers

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Proceedings of the American Society of Civil Engineers

PHYSICAL CHARACTERISTICS OF DRAINAGE BASINS

By Bernard L. Golding,[1] M. ASCE and Dana E. Low,[2] A. M. ASCE

SYNOPSIS

'he physical characteristics of major significance, such as; area of the nage-basin, slope of the principal drainage channel, length of the princi- drainage channel, shape of the drainage-basin (basin shape), and general e of the land in the basin (land slope), are reviewed and defined in terms ecent usage. A standard procedure for plotting the area-distance curves area-elevation curves is suggested. Also discussed is a new method de- ped for computation of a modified area-elevation curve. Numerical pro- res for computing the various important physical characteristics utilizing standard area-distance curves and the modified area-elevation curves are ided.

INTRODUCTION

enerally, the hydraulic engineer is lucky if there is a stream-gaging station e immediate vicinity upstream or downstream of the point in which he is ested. Thus in most cases, he is required to estimate stream flow by com- ng the drainage-basin above the point of interest with another basin having lar physical characteristics. In the case of a highway, the hydraulic engi- generally computes discharge corresponding to a particular frequency by g a graph for that frequency and entering with an argument in some way

ote.—Discussion open until August 1, 1960. To extend the closing date one month, itten request must be filed with the Executive Secretary, ASCE. This paper is part e copyrighted Journal of the Hydraulics Division, Proceedings of the American So- of Civil Engineers, Vol. 86, No. HY 3, March, 1960.

Head, Hydr. Dept., Howard, Needles, Tammen & Bergendoff, New York, N. Y.
Hydr. Engr., Howard, Needles, Tammen & Bergendoff, New York, N. Y.

expressive of the physical characteristics of the drainage-basin. These gra
have been developed by correlating discharge against physical characteris
of many similar drainage-basins. In the case of a dam, the unit-hydrogr
method is used. Coordinates of the peak value of the unit hydrograph are ta
from a graph in which the peak (maximum y-coordinate) and lag (correspon
x-coordinate) of unit hydrographs for basins having similar physical char
teristics have been correlated with the physical characteristics of those bas

Many recent papers have been published using such correlations so th
knowledge of basin characteristics has become an essential for any engin
working in the hydraulics field.

These various physical characteristics of drainage-basins have been def
in many different publications in the past by many different authors.[3,4,5,6]
has resulted in some of the same named basin characteristics being define
several different ways. Also, many of these original publications are no lo
er obtainable as they are out of print. This has made the frequent referen
to such publications in many recent papers quite useless.

In an attempt to remedy this situation, the various drainage-basin char
teristics are reviewed, and a suggested procedure for the computation of
more important characteristics is presented.

AREA OF THE DRAINAGE BASIN

The area of the drainage-basin is its most important physical charac
istic. The area is actually the horizontal projection of the land surface f
which run off into the surface channels, above the point of interest, occurs
is ordinarily measured on topographic maps by location of the divide, or se
rating ridge, which determines whether surface run off will flow toward
basin in question, or flow into adjacent basins. The size of the drainage
is generally expressed in square miles (or acres for small areas). Occas
ally, sub-surface drainage-basins (ground-water flow) encompass diffe
areas. However, this complication is very rare and generally not of g
significance.

LENGTH OF THE PRINCIPAL DRAINAGE CHANNEL

The length of the principal drainage channel is normally measured a
the main channel, from the point of interest to the drainage-basin divide.
length is normally measured as short chords on United States Geological
vey (USGS) quadrangle maps, which along with the projection of the prin
channel to the basin divide, may cause some minor discrepancies in the le
when measured by different individuals. However, such discrepancies
generally insignificant in their effect on hydrologic computations.

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[3] "Topographic Characteristics of Drainage Basins," by W. B. Langbein, U. S.
Survey Water Supply Paper 968-C, 1947.

[4] "Drainage Basin Characteristics," by R. E. Horton, Transactions Amer. Geop
cal Union, Vol. 13, 1932, pp. 350-361.

[5] "Synthetic Unit Graphs," by F. F. Snyder, Transactions Amer. Geophysical U
Vol. 19, Part 1, 1938, pp. 447-454.

[6] "Unit-Hydrograph Lag and Peak Flow Related to Basin Characteristics," by
Taylor and H. E. Schwarz, Transactions Amer. Geophysical Union, Vol. 33, 195
235-246.

SLOPE OF THE PRINCIPAL DRAINAGE CHANNEL

The slope of the principal drainage channel is probably the most significant ysical characteristic after the area of the drainage-basin. The simplest thod for expressing the slope of the principal drainage channel is to divide length of the channel by the difference in elevation between its upper and ver ends (Definition 1, Fig. 1). Generally, the length of the channel is mea-red beyond the upper end of the clearly discernible stream channel to the inage divide and then divided by the difference in elevation between this nt on the ridge line and the point of interest on the channel. The USGS has ed a slope parameter in studies in the New England states in which the length that part of the stream between points 85% and 10% of the total distance above point of interest is divided by the difference in elevation at these places.[7] ey are of the opinion that the most upstream part of the slope, in the steep

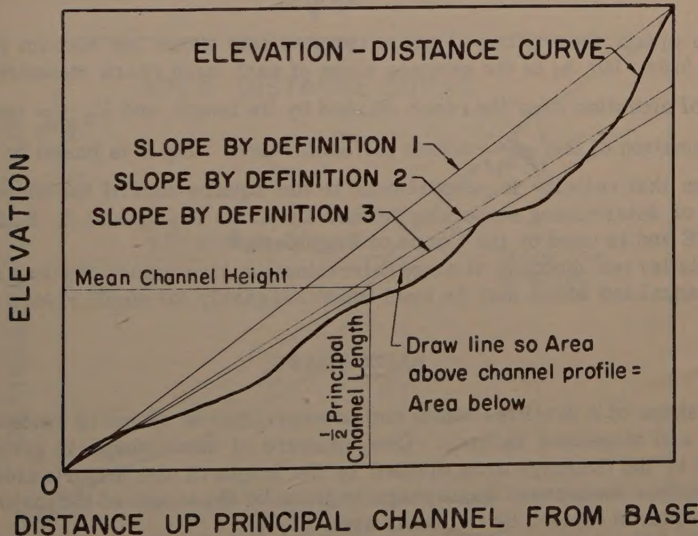

FIG. 1.—SLOPE OF THE PRINCIPAL DRAINAGE CHANNEL

adwaters, might affect the slope out of proportion to the volume of water fur-shed by the headwater area, and also that the most downstream part of the ope of the principal channel may be so flat that it might not indicate the true rtion of the flow which the lower portion of the drainage area actually does ntribute.

A slope of the principal drainage channel can also be determined by dividing ice the mean height of the channel profile above the point of interest by the ngth of the principal channel.

The mean height of the profile is obtained by integration of the elevation-stance curve (channel profile) and dividing by the length of the channel (Defi-ion 3, Fig. 1). The integration can be accomplished by "planimetering" the ea below the elevation-distance curve. This slope is also given by the slope

7 "Channel-Slope Factor in Flood Frequency Analyses," by M. S. Benson, Proceed-s Paper 1994, Journal of the Hydr. Div., ASCE, Vol. 85, No. HY 4, April, 1959, pp. .

of the line drawn through the origin of the stream-profile curve such that th area under it equals the area under the profile curve. The mean channel heigh is at a point on the above-cited line at one-half the length of the principal drain age channel measured from either end. Rather than computing mean channe height as such for use in determining channel slope, the mean basin height (t be defined, subsequently, under the heading "Land Slope") is often used and th resultant slope is called the mean basin slope. This effectively introduces weighting factor to account for slope of the basin as well as slope of the strea channel.

A probable better and more rational definition for slope of the princip drainage channel is the slope of an equivalent stream having the same trav time and same length (Definition 2, Fig. 1). This slope is computed by

$$S_{st} = \frac{p}{\Sigma p \frac{1}{\sqrt{s_i}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots ($$

where p equals the number of equal reaches into which the stream has bee divided (often 10), s_i is the average slope of each such reach measured as t change of elevation over the reach divided by its length, and $\Sigma_p \frac{1}{\sqrt{s_i}}$ represen the summation of the $\left(\frac{1}{\sqrt{s_i}}\right)$-values for each reach. Eq. 1 is based on the as sumption that velocity is proportional to the square root of the slope. Th method of determining slope was defined by A. B. Taylor and H. E. Schwar M. ASCE and is used by the Corps of Engineers.[8]

The latter two methods of slope determination give values that are approx mately equal and which may be used interchangeably for small rivers (Fig. 1

BASIN SHAPE

The shape of a drainage-basin can be expressed in terms of various eas defined and measured factors. One measure of basin shape is given by t quotient of the drainage area divided by the length of the major watercours A/L. Another measure of basin shape is given by the length of the major wate course squared divided by the basin area L^2/A.

The most commonly used basin shape factors are L_{ca} and the product times L_{ca} to some power, usually $(L\ L_{ca})^{0.30}$, where L is the length along t longest watercourse and L_{ca} is often defined as the distance along the ma drainage channel from the point of interest to a point opposite the comput center of gravity (centroid) of the drainage area.[5,6] A more accurate definiti of L_{ca} is given subsequently. The term L_{ca} is computed by integration of t area-distance curve and dividing by the drainage area.

To construct the area-distance curve (Fig. 2) the channel length is first su divided into separate reaches. These reaches may be of equal length along t watercourse, such as 1-mile reaches or equal parts of the total length. Divi ing points along the stream could also be made at points where contours cro A preferable method, especially for larger rivers, is to divide the watercou length just above and below each major tributary. The areas in this case a the contributary drainage areas at these points. After the channel is divic into reaches, the sub-drainage basins contributing to the stream between limits of each reach are determined.

[8] Unit Hydrograph Compilations, Project CW 153, Washington District, Corp Engrs., Washington, D. C., 3 Volumes 1949, 1 Volume 1954.

A curve of channel distance upstream from the point of interest (or per-
tage of channel distance) versus area (or percentage of area) contributing
hin that distance is then drawn as shown in Fig. 2. The distance to the cen-
id of the area is found by dividing the area above the curve by the drainage
a (or percentage of the drainage area) which is the maximum ordinate. As
alternative, L_{ca} may also be found by integrating the area-distance curve
dividing by the drainage area. The integration is accomplished by "plani-
tering" the area above the area-distance curve.

A suggested method for plotting the area-distance curve and a sample com-
ation of L_{ca} based on this curve will be presented subsequently.

Physical Definition of L_{ca}.—Previously, it was stated that the centroid of
area above the area-distance curve determines L_{ca}. In other words, L_{ca}
the length measured up the stream channel from the base of the drainage
a to a point corresponding to the centroid of the area above the area-distance

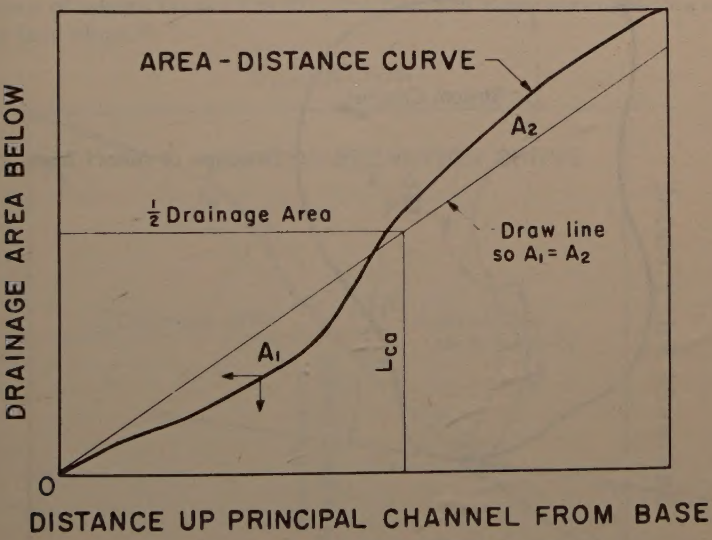

FIG. 2.—BASIN SHAPE FACTOR, L_{ca}

ve. Although all of the various methods of approach that utilize the L_{ca}
ction derive it in the same general manner, some writers have added physi-
definitions of L_{ca} which are not rigidly correct while others leave the
sical interpretation of L_{ca} purposely vague. For example, L_{ca} has been
ined as "the distance along the stream channel to a point opposite the cen-
id of the drainage area."[6] This definition is actually incorrect since the lo-
ion of the centroid of an area depends solely on its shape, and L_{ca}, derived
m the area-distance curve, obviously depends on other factors as well. On
other hand, L_{ca} is sometimes vaguely defined as "the distance to the center
rea."[9]

Physically speaking, just what is L_{ca}? Fig. 3 shows a hypothetical drainage
a. Axis x-x is drawn perpendicular to the major axis of the area through

---

[9] "Applied Hydrology," by R. K. Linsley, M. A. Kohler, and J. L. H. Paulhus,
Graw-Hill, New York, 1949, p. 456.

the base of the area. The true area-centroid is then located a distance $\bar{y}$ abov this axis where

$$\bar{y} = \frac{\Sigma^A \; y_1 \; \Delta A}{A} \qquad \text{(2}$$

in which ΔA is an elemental area, y_1 is its distance from the axis x-x, and is the total area. The term L_{ca} can be expressed by a similar formula

$$L_{ca} = \frac{\Sigma^A \; y_2 \; \Delta A}{A} \qquad \text{(3}$$

in Eq. 3 y_2 is the distance measured along the stream to the point on the chan nel at which overland runoff resulting from rainfall on ΔA first reaches th stream.

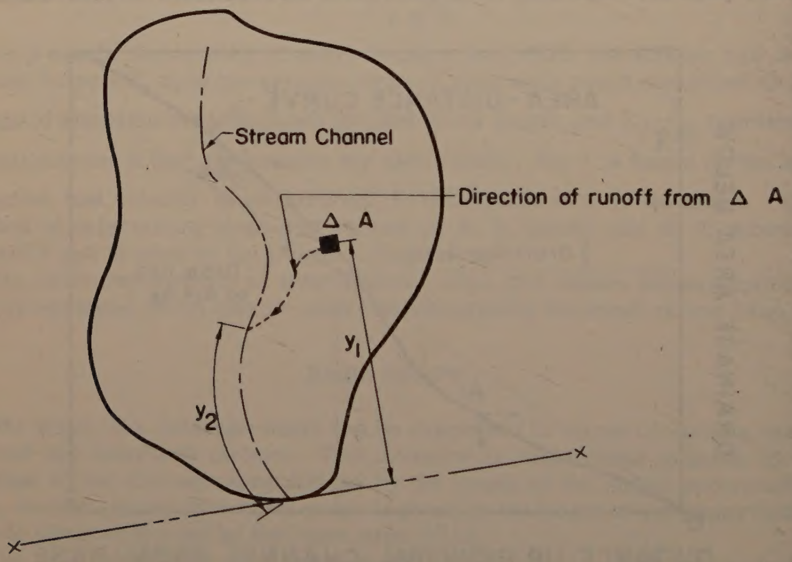

FIG. 3.—HYPOTHETICAL DRAINAGE AREA

LAND SLOPE

Defining a characteristic which adequately describes land slope has prove difficult. However, this characteristic is a major factor in indicating the trave time of overland flow, so several methods for determining land slope have bee developed.

Most of the methods generally used for defining land slope take into accou the general slope of the entire basin including the tributary stream netwo contributing to the major watercourse, but excluding the major watercourse.

Average land slope has been defined as equal to the plotted contour interv times the total length along all contours in the drainage-basin divided by th drainage area. However, measuring the length of contours proved a tediou and time-consuming operation and hence the following short-cut methods we derived and are used:

In the intersection-line method, a grid of uniformly spaced parallel and rpendicular lines is laid over a contour map of the drainage area. The num- r of contours crossing each subdivision of the grid within the area is counted. e land slope in either grid direction is then $s = \frac{N \, \Delta Z}{L}$ where N is the total mber of contour crossings for all lines in one direction, ΔZ denotes the con- r interval, and L is the total length of grid lines in one direction within the ea. The average land slope is then given by some relation between these two ues of slope or by

$$s' = \frac{N' \, \Delta Z}{L'} \sec \theta \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (4)$$

ere N' and L' are the sum in both grid directions of the values previously ined as N and L, and θ is the average angle of intersection between the con- rs and grid lines. Often a value of 1.57 is used for sec θ as it is the av- ge secant of angles from 0° to 90°. The Corps of Engineers uses this method inding land slope.[8]

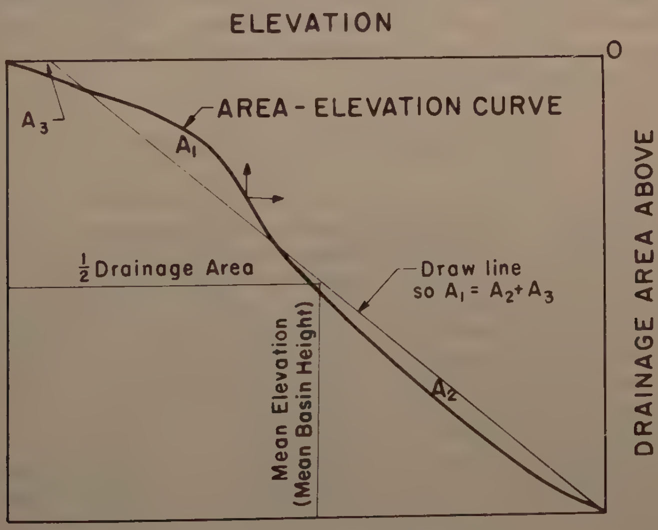

FIG. 4.—LAND SLOPE

Another method of defining land slope, or more accurately, basin slope, util- s an area-elevation curve in which the drainage area above a certain ele- on is plotted against that elevation. Area-elevation data can be assembled planimetering the area enclosed by each contour and the basin divide and ted as shown on Fig. 4.

The mean basin height can be obtained by integration of the area-elevation ve and dividing the result by the drainage area. It can also be obtained by method shown in Fig. 4 in which a line is drawn such that $A_1 = A_2 + A_3$. intersection of this line with a line drawn parallel to the abscissa at 50% of drainage area indicates the mean basin height. Mean basin slope is then ned as two times the mean basin height divided by the length of the channel.

Another curve can be plotted conveniently by using the same data derive for producing an area-distance curve. This is called a modified area-elevatic curve. Rather than cutting the drainage-basin into slices between adjacent cor tours, it can be divided into sub-drainage basins contributing to equal reache along the length of the principal drainage channel to correspond with the metho used in computing the area-distance curve. The preferable method, especiall for larger rivers, is to break the channel at the tributaries just above the poi where they join the main channel as described under the heading "Basin Shape. In this case, area no longer refers to area lying above a certain elevation b rather to the area contributing runoff to the major stream above the point which the stream bed is at a certain elevation. Thus the mean basin height obtained actually weighted by distribution of drainage area long the stream pro file. Mean elevation and mean basin slope are correspondingly weighted.

The foregoing method saves considerable computation time over the usu method for computing an area-elevation curve by utilizing one set of area da as the basis for both curves. In the opinion of the writers this results in more meaningful mean basin height and mean basin slope. It is undoubted not applicable in studies of snow hydrology where the temperature-elevatic curve makes true land elevations more significant. Further study may al indicate that the proposed modification should not be used in certain other su divisions of the subject of hydrology.

SUGGESTED PROCEDURE FOR DEVELOPING CURVES

In the plotting of the area-elevation curve to compute mean elevation a mean basin height and the area-distance curve to compute L_{ca}, it makes difference whether zero channel distance is taken at the base of the draina area or at the drainage divide. Also it does not really matter whether t drainage area above a certain point on the channel or the drainage area bel that point is plotted versus that channel distance. The plotting units may actual or percentage values and the location and sense of scales could co ceivably vary in any one of several ways. However, in order to eliminate a possible confusion a suggested standard procedure similar to the method us by the Corps of Engineers[8] is outlined herewith:

Channel distance will always be measured up the channel from the base a will be expressed in percentage of total length. Elevation will always be me sured above the stream-bed elevation at the base of the drainage-basin and w be expressed as the percentage of the total elevation drop from one end of t stream to the other. Drainage area will also be expressed as a percentage the total. It will be recalled that in the development of the area-distance r lationship the term area referred to is the area of the drainage-basin contribu ing below the point along the stream in question. On the other hand the are elevation relationship required the consideration of the contributary area abo the point in question. This distinction need cause no confusion in actual pra tice, as will be shown.

The fact that all scales go from 0% to 100% enables the engineer to plot bo curves on a square piece of graph paper. Since both curves appear roughly diagonals on such a graph, the scales are oriented so that they will cross o once, thus eliminating any danger of confusion (Fig. 5). The value of L_{ca} then found by measuring the area above the area-distance curve in square inc es and converting to square percentage, for example, for horizontal and ver

scales of 1 in. = 10%, the conversion factor is 1 sq in. = $(100\%)^2$. This ure divided by 100% is the L_{ca} expressed as a percentage of the total length.

To find mean basin height (mean elevation), the area above the area-elevation rve is determined, converted to square percentage and divided by 100%. The sult is the mean basin height expressed as a percentage of the total height. an basin slope is then mean basin height divided by one-half the total chan- length, both values having been converted from percentage to actual lengths.

The modified area-elevation curve is plotted in a similar fashion to that de- ribed for the standard area-elevation relationship.

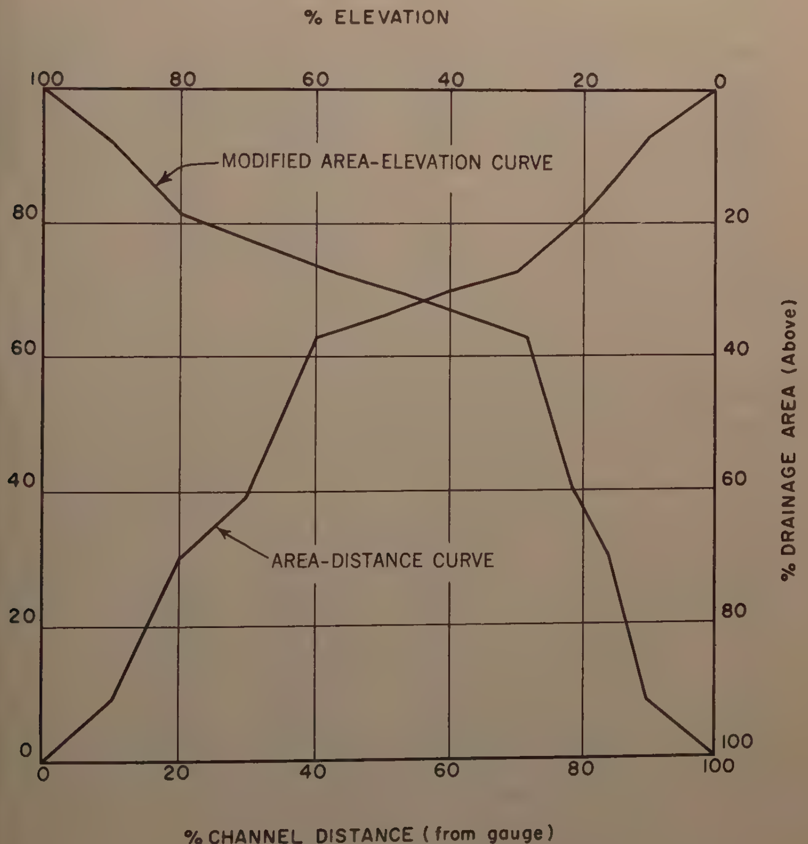

FIG. 5.—HYPOTHETICAL DRAINAGE BASIN

Example.—The necessary points for plotting a modified area-elevation curve l an area-distance curve for a hypothetical drainage-basin are shown in ble 1. Sub-divisions on the stream were made at even tenth points along the al channel length. Other methods, as previously noted, could have been used. e that distance is measured in an upstream direction and that area and ele- ion are measured below the point on the channel. The modified area- vation curve is plotted using the lower right-hand corner as the origin.

An alternate solution is to add another column to Table 1 for percentage area above, which is determined by subtracting percentage area below from 100%. The modified area-elevation curve can then be plotted directly using the origin in the upper-right corner.

TABLE 1.—HYPOTHETICAL DRAINAGE BASIN (DRAINAGE AREA = 8.10 SQ MILES, CHANNEL LENGTH TO BASIN DIVIDE = 5.42 MILES)

Length, L, in feet (1)	% L (2)	Elevation, E, in feet (3)	% E below (4)	Area, A, in square miles (5)	% A below (6)
0	0	21	0	0	0
2,860	10	43	11	0.70	8.6
5,720	20	54	16.5	2.43	30.0
8,590	30	63	21	3.14	38.8
11,450	40	78	28.5	5.07	62.6
14,310	50	94	36.5	5.28	65.2
17,170	60	116	47.5	5.62	69.4
20,030	70	134	56.5	5.84	72.1
22,900	80	180	79.5	6.55	80.8
25,760	90	202	90.5	7.51	92.7
28,620	100	221	100	8.10	100.0

TABLE 2

Elevation, E, in feet (1)	E (2)	Reach length, in feet (3)	Reach slope, S_i (4)	$\sqrt{S_i}$ (5)	$\frac{1}{\sqrt{S_i}}$ (6)
21					
43	22	2,860	0.00770	0.0878	11.4
54	11	2,960	0.00385	0.0621	16.1
63	9	2,860	0.00315	0.0561	17.8
78	15	2,860	0.00525	0.0725	13.8
94	16	2,860	0.00560	0.0748	13.4
116	22	2,860	0.00770	0.0878	11.4
134	18	2,860	0.00630	0.0794	12.6
180	46	2,860	0.01608	0.1270	7.9
202	22	2,860	0.00770	0.0878	11.4
221	21	2,860	0.00735	0.0858	11.7
					127.5

$$S_{st} \left(\frac{10}{127.5}\right)^2 = 0.00615 \text{ ft per ft}$$

A sample computation of the weighted values of mean basin height, mean elevation and land slope (mean basin slope) based on the data given in Table is as follows:

Area above modified area-elevation curve = (3810%)²
Mean Basin Height = (0.381) x (221 - 21) = 76.2 ft
Mean Elevation = 21 + 76.2 = 97.2 ft (m.s.l.)
Land Slope (Mean Basin Slope) = 76.2/14,310 = 0.00532 ft per ft
= 28.1 ft per mile

he computation of L_{ca} is as follows:

Area above area-distance curve = $(4330\%)^2$
$L_{ca} = (0.433) \times (5.42) = 2.34$ miles

etermination of the Channel Slope.—In Table 2 stream slope for the same thetical drainage basin considered in Table 1 is computed by the method iously reviewed in which the slope of the principal drainage channel is exsed as the slope of an equivalent stream having the same travel time and length.[6,8] Pertinent information from Table 1 is thus repeated in Table 2.

SUMMARY

review of the more important physical characteristics of drainage-basins een presented along with a suggested standard procedure of plotting the rtant area-distance curve, from which L_{ca} can be computed, and the areation curve from which the land slope (mean basin slope) can be computed. ditional characteristic, developed from the modified area-elevation curve, een described.

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DEVELOPMENT OF FLOW IN TANK DRAINING

By David Burgreen[1]

SYNOPSIS

theoretical study is made of the development of velocity with time when ve at the end of a drain pipe is suddenly opened. The effect of friction, surization, and length of drain pipe is examined. The true velocity, obd by considering the fluid inertia, is compared to the velocity obtained by nore well-known method which disregards the fluid inertia and assumes the draining is governed only by the prevailing head.

INTRODUCTION

ne problem treated herein is a generalization of a practical problem ng with the shutdown of a reactor by the rapid draining of moderator or ctor liquid. It is important to know quantitatively the rate of draining of iquid from the tank to ascertain that it is sufficiently high to give rapid tor shutdown. The mechanics of rapid liquid-level change or of rapid tank ing falls in the realm of non-steady flow. The moderator or reflector l is initially at rest, and the order of magnitude of time required to reach

te.—Discussion open until August 1, 1960. To extend the closing date one month, ten request must be filed with the Executive Secretary, ASCE. This paper is part copyrighted Journal of the Hydraulics Division, Proceedings of the American Soof Civil Engineers, Vol. 86, No. HY 3, March, 1960.

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peak velocity is the same order of magnitude of time, from a nuclear point
view, in which the shutdown must be completed.

The more well-known type of tank-draining problem is the one in which
rate of discharge is governed by the decreasing head in the tank. Althou
the rate of flow varies with time, this is not a true non-steady flow proble
but rather one of slowly varying steady flow. In flow under decreasing he
the inertia forces are neglected, whereas in non-steady flow the presence
inertia forces is implied. The flow relationships that are derived without
inclusion of inertia forces are designated as "terminal flow" relationships
this paper, since they represent an approximation of the terminal phase of
flow.

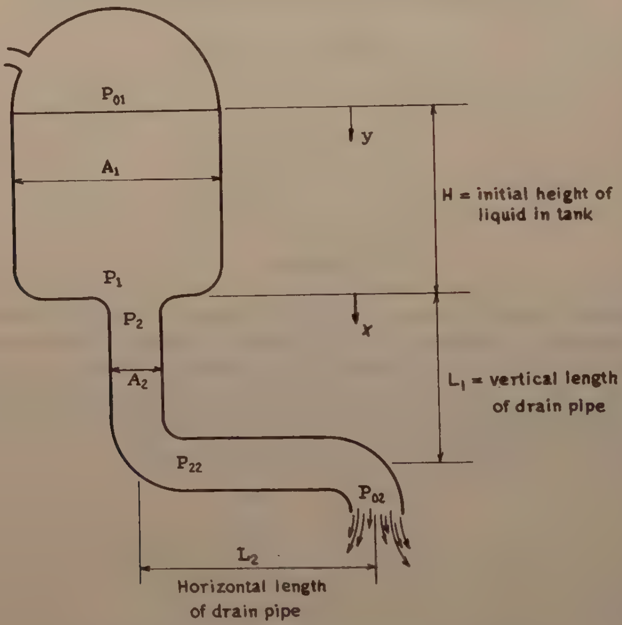

FIG. 1.—TANK AND DRAIN PIPE ARRANGEMENT

A related problem dealing with the hydrodynamics of a conservative m
and energy system has already been studied.[2] The present problem d
with a non-conservative mass and energy system, in which mass and en
in the form of draining liquid is continuously being lost from the system.

BASIC EQUATIONS

The system consists of a large pressurized tank from which liquid dr
through a drain pipe of smaller diameter having a vertical and a horiz
length. It drains under the influence of gravity, assisted in some cases
gas pressure on the surface of the liquid. Fig. 1 shows the arrangeme

[2] "Hydrodynamics of Liquid Poison Scram System," by David Burgreen, Nu
Science and Engineering, Vol. 4, No. 1, July 1958.

nd drain pipe. The summation of forces on the slug of liquid in the tank

$$_1 A_1 - P_1 A_1 + w\,(H-y)A_1 - \frac{w}{g}\,(H-y)\,A_1 \frac{d^2y}{dt^2} = 0 \quad \ldots\ldots\ldots\ldots\ldots (1)$$

is assumed in deriving Eq. 1 that friction forces retarding the flow in nk may be neglected. A balance of forces on the slugs of liquid in the cal and horizontal lengths of drain pipe give, respectively:

$$_{22} A_2 + P_2 A_2 + w A_2 L_1 - \frac{w}{g} L_1 A_2 \frac{d^2x}{dt^2} - \frac{w}{2\,g} A_2 f_1 \frac{L_1}{D_2}\left(\frac{dx}{dt}\right)^2 = 0 \ldots (2a)$$

$$_2 A_2 - P_{02} A_2 - \frac{w}{g} L_2 A_2 \frac{d^2x}{dt^2} - \frac{w}{2\,g} A_2 f_1 \frac{L_2}{D_2}\left(\frac{dx}{dt}\right)^2 = 0 \quad \ldots\ldots\ldots (2b)$$

ich x represents the displacement of a fluid particle in the drain pipe s most clearly defined by the continuity relationship $A_1 y = A_2 x$. The on of Eqs. 2 yields

$$_{02} + P_2 + w L_1 - \frac{w}{g} L \frac{d^2x}{dt^2} - \frac{w}{2\,g} F \left(\frac{dx}{dt}\right)^2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

q. 3, $L = L_1 + L_2$ and F represents the number of velocity heads lost gh pipe friction and would include entrance, exit, bend, and valve losses dition to the straight pipe losses. At the junction of the pipe and tank the ure-velocity relationship is the same as for steady flow.[2] It is

$$- P_2 = \frac{w}{2\,g}\left(\frac{dx}{dt}\right)^2 \left[1 - \left(\frac{A_2}{A_1}\right)^2\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

ded the length of the drain pipe is greater than its diameter.
e use of the continuity relationship, $A_1 y = A_2 x$, together with Eqs. 1, 3, produces:

$$\frac{A_2}{A_1} H - \left(\frac{A_2}{A_1}\right)^2 x\Big] \frac{d^2x}{dt^2} + \frac{1}{2}\left[1 + F - \left(\frac{A_2}{A_1}\right)^2\right]\left(\frac{dx}{dt}\right)^2$$

$$- g\left[H + \frac{P_{01}}{w} + L_1 - \frac{A_2}{A_1} x\right] = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

. 5 is expressed in terms of the displacement of a particle in the drain The flow could be expressed just as readily in terms of the drop of level tank, y. In deriving Eq. 5, P_{02}, the pressure at the end of the drain pipe een assumed to be atmospheric (zero gage).

INITIAL ACCELERATION

the start of flow the displacement and the velocity are zero. From the initial acceleration is

$$\left(\frac{d^2x}{dt^2}\right)_{t=0} = g\left[\frac{(n+1)\,H + L_1}{L + R\,H}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

where $R = A_2/A_1$ and $n\ H = P_{01}/w$. The pressurization in the tank is thus pressed as the number, n, of initial tank heads. The initial acceleration of liquid in the tank, which is the same as the acceleration of the dropping s face of liquid is

$$\left(\frac{d^2y}{dt^2}\right)_{t=0} = R\ g \left[\frac{(n+1)\ H + L_1}{L + R\ H}\right] \ldots\ldots\ldots\ldots\ldots$$

It is clear from Eq. 7 that without pressurization the initial acceleratior the liquid in a tank, to which a drain pipe is attached, can never be as high g, since $R < 1$. In order to obtain an initial acceleration of g, the requi pressurization n H is

$$n\ H = \frac{L_2 + (1-R)\ L_1}{R} \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

CROSS-SECTIONAL AREA OF A VERY SMALL DRAIN PIPE

When the flow area of the drain pipe is very small in comparison to flow area of the tank, Eq. 5 simplifies to

$$L\ \frac{d^2x}{dt^2} + \frac{1+F}{2}\left(\frac{dx}{dt}\right)^2 - g[\ (n+1)\ H + L_1\ ] = 0 \ldots\ldots\ldots$$

and represents flow under constant head with inertia effects. It is thus indi tive of the initial flow behavior only, since the approximation of Eq. 9 is valid for large x. The first integrals of Eq. 9 with respect to time and placement are, respectively,

$$\frac{dx}{dt} = \left[\frac{2\ g[\ (n+1)\ H + L_1]}{1+F}\right]^{1/2} \tanh\left[\{2\ g[\ (n+1)\ H+L_1]\ (1+F)\}^{1/2}\ \frac{t}{2\ L}\right] \ldots\ ($$

and

$$\frac{dx}{dt} = \left[\frac{2\ g[\ (n+1)\ H+L_1]}{1+F}\right]^{1/2} \left[1-e^{-[\ (1+F)/L]x}\right]^{1/2} \ldots\ldots\ldots\ ($$

and the displacement may be written explicitly as a function of time by co bining Eqs. 10:

$$x = \frac{L}{1+F}\ \ln\left[1 - \tanh^2\left(\{2\ g[\ (n+1)H+L_1]\ (1+F)\}^{1/2}\ \frac{t}{2L}\right)\right]^{-1} \ldots\ldots$$

If there were a horizontal drain pipe only and no pressurization or fric then Eq. 10a would become

$$\frac{dx}{dt} = [\ 2\ g\ H+1]^{1/2} \tanh\left[(2\ g\ H)^{1/2}\ \frac{t}{2\ L_2}\right] \ldots\ldots\ldots$$

Eq. 12 has been derived[3] from the Bernoulli equation for non-steady flow.

From Eq. 10b it is seen that for small values of x/L the velocity in drain pipe may be approximated by

$$\frac{dx}{dt} \cong \left[\frac{2\ g\ x[\ (n+1)H+L_1]}{L}\right]^{1/2} \ldots\ldots\ldots\ldots\ldots$$

---

[3] "Fundamentals of Hydro and Aeromechanics," by Prandtl-Tietjens, McGraw I

result is compatible with Eqs. 6 and 7 and is the velocity associated with stant initial acceleration. When the drain pipe area is extremely small, hange in head is very slow and the starting velocity of the terminal flow t sensitive to fluid movement in the lower drain pipe, and can be exed as

$$\left(\frac{dx}{dt}\right)_{R\to 0} = \left[\frac{2\,g[(n+1)\,H+L_1]}{1+F}\right]^{1/2} \quad \ldots\ldots\ldots\ldots\ldots (14)$$

TERMINAL FLOW

e solution of Eq. 5 with the inertia term $d^2x/dt^2 = 0$ is the solution of the known tank-draining problem in which the discharge is governed by the entary head and the inertia of the fluid to a change in velocity is disred. It is of considerable practical importance and, as will be shown, it is y close approximation of the flow for the case when both the area ratio all and the length of drain pipe is small. Under zero inertia conditions, olution of Eq. 5 in terms of the y-coordinate, or liquid level in the tank, is

$$\frac{dy}{dt} = R\left[\frac{2\,g[(n+1)H+L_1-y]}{1+F-R^2}\right]^{1/2} \quad \ldots\ldots\ldots\ldots\ldots (15)$$

ntegral of Eq. 15 gives the time required for a specified drop in liquid It is

$$\frac{1}{R}\left[\frac{2}{g}(1+F-R^2)\right]^{1/2}\left\{[(n+1)H+L_1-y_0]^{1/2}-[(n+1)H+L_1-y]^{1/2}\right\} \quad \ldots (16)$$

e y_0 is the liquid level at $t = 0$.

GENERAL SOLUTION

though Eq. 5 is a non-linear differential equation with variable coeffis, the first integral, in terms of the tank level y, is readily obtained. It the velocity as

$$= \left\{\frac{2\,g\,R^2}{1+F-R^2}\left[(n+1)H+L_1+\frac{R(L+R\,H)}{1+F-2\,R^2}\right]\left[1-\left(1-\frac{R\,y}{L+R\,H}\right)^{(1+F-R^2)/R^2}\right] - \frac{2\,g\,R^2\,y}{1+F-2\,R^2}\right\}^{1/2} \quad \ldots\ldots\ldots\ldots (17)$$

7 has four parameters in addition to the independent variable y/H. The of these parameters will be examined separately. Consider first the on when the drain pipe lengths $L_1 = L_2 = H$, when there is no pressuri, and the friction forces are neglected. Eq. 17 becomes

$$= \frac{R^2(2+2\,R-3\,R^2)}{(1-R^2)(1-2\,R^2)}\left[1-\left(1-\frac{y}{H}\,\frac{R}{2+R}\right)^{(1/R^2)-1}\right]-\frac{y}{H}\,\frac{R^2}{1-2\,R^2} \quad \ldots\ldots (18)$$

8 is plotted in Fig. 2 for several ratios of drain pipe area to tank area. this ratio is unity, the configuration represents a pipe elbow of uniform

diameter whose vertical length is twice the horizontal length. Eq. 18 t
more simply expressed as

$$\frac{V^2}{2\,g\,H} = \frac{y}{H} + \ln\left(1 - \frac{y}{3\,H}\right),\; y < 2H \ldots\ldots\ldots\ldots\ldots\ldots$$

A more general expression derived from Eq. 17, neglecting friction and pr
surization, for the discharge of fluid from a pipe elbow of uniform diame
of total length L + H and horizontal length $L_2 = L - L_1$ is

$$\frac{V^2}{2\,g\,H} = \frac{y}{H} + \frac{L_2}{H} \ln\left(1 - \frac{y}{H}\;\frac{H}{L+H}\right),\; y < H + L_1 \ldots\ldots\ldots\ldots\ldots\ldots($$

In Fig. 2, a free-fall curve, which is included for comparative purpos
shows the velocity of a free-falling body, dropped from a height H, at any e

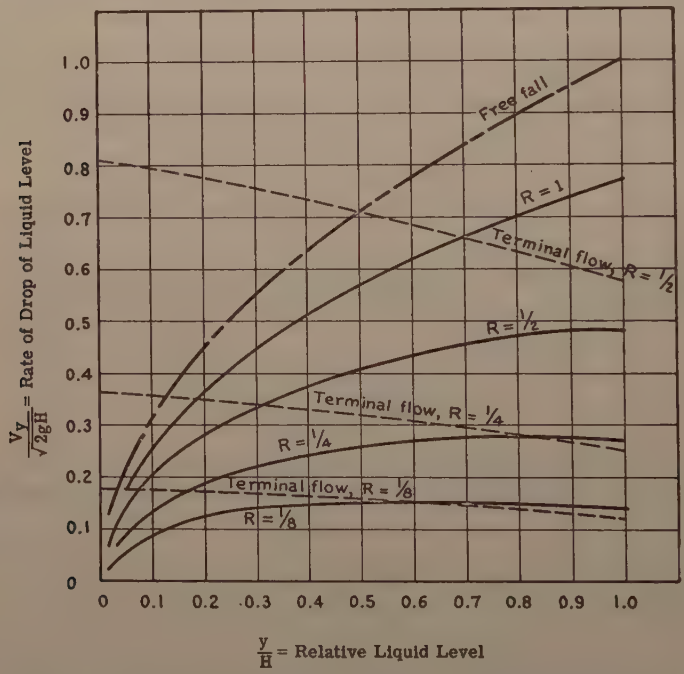

FIG. 2.—VARIATION OF FREE SURFACE VELOCITY WITH LIQUID LEVEL IN TANK. $L_1 = L_2 = H$; n = 0; F = 0.

vation. It is a special case of Eq. 20, or the R = 1 curve when L_2, the ho
zontal length of pipe, is equal to zero. The curves in Fig. 2 show the rat
fall of the liquid surface in the tank to decrease with decreasing area rat
The range in which the flow is governed primarily by inertia forces also
comes smaller with decreasing area ratio. This may be observed through
increasingly early blending of the true flow curves with the terminal f
curves as the area ratio becomes smaller. The dotted terminal flow cur
are plotted from Eq. 15 which neglects inertia effects. These are fair
proximations of the terminal stages of the flow.

ıen the area ratio is quite large as, for example, when R = 1/2 the ity-displacement curve shows that the inertia forces primarily deter- the character of the flow. For large area ratios the velocity- ıcement curves resemble the free-fall curve and any approximation with nal flow would not be valid. Note that the true velocity-displacement s indicate, properly, that the velocity starts at zero and builds up, while erminal flow curves indicate, incorrectly, that at the start of flow the ity is $V = (2\,g\,H)^{1/2}$.

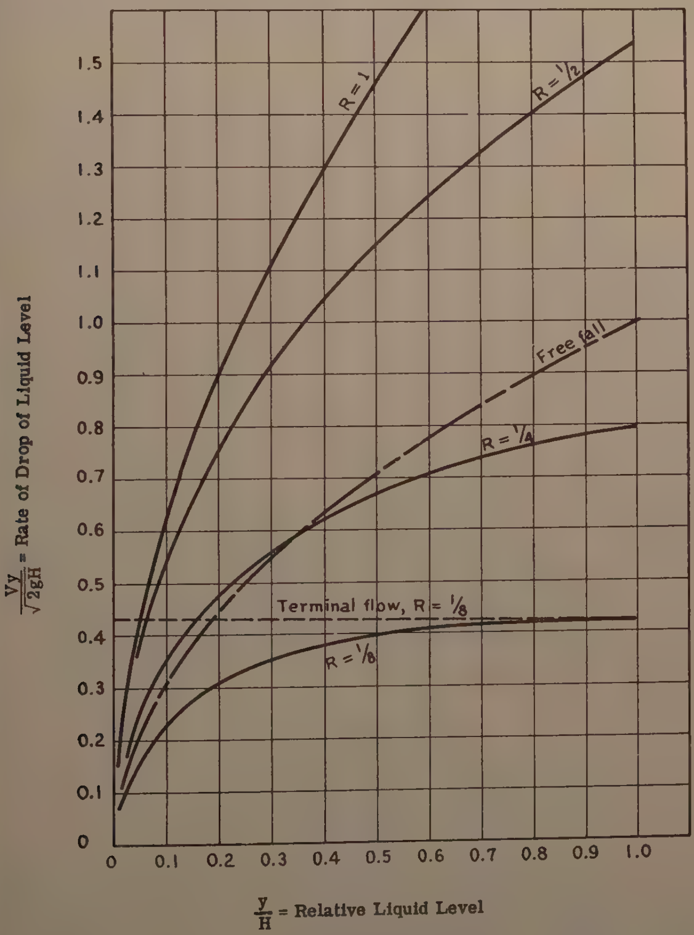

FIG. 3.—VARIATION OF FREE SURFACE VELOCITY WITH LIQUID LEVEL IN TANK; PRESSURIZATION: $n=10$; $F=0$; $L_1=L_2=H$.

e effect of pressurization is shown in Fig. 3. The pressure on surface liquid is assumed to be constant and is arbitrarily taken as ten tank ; that is, n = 10. These curves show that a rate of drop in liquid level, ess of the free-fall rate is easily obtained. It will be observed that for elected pressurization, an area ratio of 1/4 produces a rate of drop in

liquid level approximating free fall. The terminal flow approximation is r
rather poor even for small area ratios.

The result of taking friction into account is demonstrated in Fig. 4. '
total friction loss in the system is arbitrarily taken at three velocity he
(drain-pipe velocity). Comparing the curves of Fig. 4 with those of Fig
and 3 it will be noted that the velocities, for corresponding area ratios,
smaller and that terminal flow is established sooner. Although terminal f
is now a better approximation to true flow, it is obviously not a satisfact
approximation, for moderate area ratios, of the early stage flow.

The effect of drain pipe length on the development of flow is well dem
strated by considering a configuration having a small area ratio typical

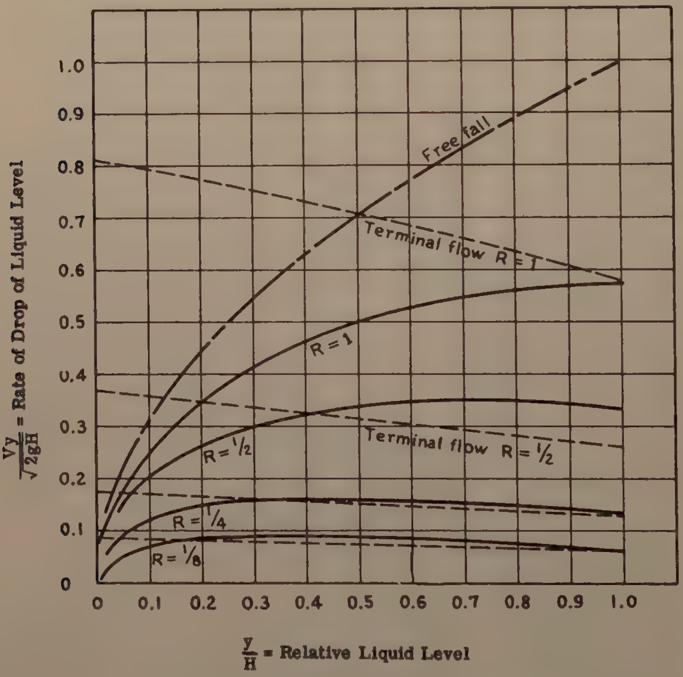

FIG. 4.—VARIATION OF FREE SURFACE VELOCITY WITH LIQUID LEVEL IN TANK. $L_1 = L_2 = H$; $n = 0$; $F = 3$.

water tank with drain pipe attachment. An area ratio of 0.01 is selected,
the velocity-displacement relationship is plotted in Fig. 5 for lengths of d
pipe of zero, H, and 5 H. The curves show, in common, that the ine
transient is completed and terminal flow started when the maximum rat
flow, more or less, has been attained.

The curve in Fig. 5a represents the development of flow when a very
drain pipe, equal to 5 H, is attached to the tank. The peak velocity is not
tained until about half the tank has been drained. This indicates that a
drain pipe produces a long inertia transient, and that computations of
charge based on the prevailing head lead to incorrect results — even when

ratio is small. When the drain pipe is of moderate length, equal to H, the peak velocity is reached when one-tenth the tank has been drained. is shown in Fig. 5b.

e fundamental differences between inertia flow and terminal flow are ht out by the curves in Fig. 6 in which the fluid acceleration is plotted st the drop in liquid level. Since terminal flow is flow without accelera- the deviation from terminal flow is measured by the magnitude of the eration. The parameters are the same as those used in the velocity- acement curves of Fig. 2; namely, unpressurized frictionless flow with $L_2 = H$. The uppermost curve is a straight horizontal line representing

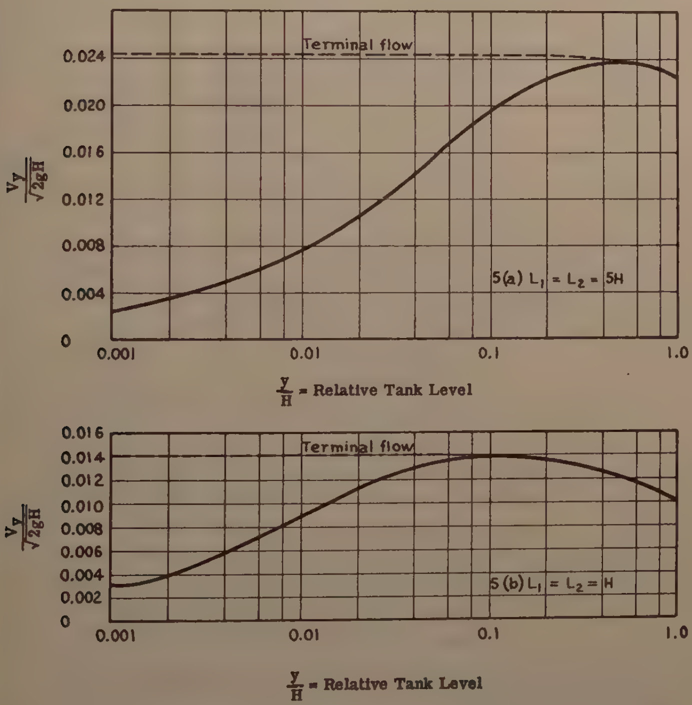

FIG. 5.—DEVELOPMENT OF FREE SURFACE VELOCITY WITH LIQUID LEVEL IN TANK. $R = 1/100$; $n = 0$; $F = 0$.

fall, or the equation $(1/g)(d^2y/dt^2) = 1$. An examination of the curves will that for each case the acceleration is a maximum at the start of flow hereafter decreases continuously with increasing displacement. As the ratio becomes small the curves show a range of decelerating flow which esponds to the region of decreasing velocity. Terminal flow is repre- d by the abscissa, $(1/g)$ $(d^2y/dt^2) = 0$. The general trend of the curves vard zero acceleration as the area ratio approaches zero. This general will prevail also for configurations other than $L_1 = L_2 = H$, but with r drain pipes will tend toward zero acceleration at a slower rate.

To obtain time-displacement curves it is necessary to integrate Eq. Analytically, it appears to be possible to perform the integration only for s cific values of F and R. In general the time-displacement curve would be

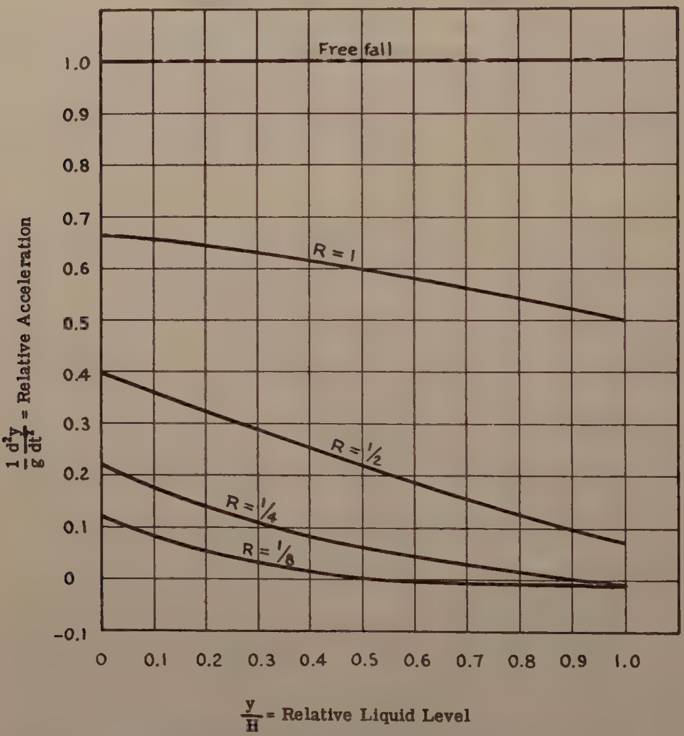

FIG. 6.—VARIATION OF ACCELERATION OF FREE SURFACE WITH LIQUID LEVEL IN TANK: $L_1 = L_2 = H$; $n = 0$; $F = 0$.

tained by numerical integration of the velocity-displacement curve. The t the reach a given level, y_n, is

$$t_n = n\,\Delta y \sum_{i=1}^{i=n} \frac{1}{V_i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Initially 1/V tends to infinity. This represents a difficulty for numerical tegration. For small values of y/H Eq. 17 becomes

$$V_y^2 \cong \frac{2\,g\,y\,R[(n+1)H+L_1]}{L+R\,H}, \quad \frac{y}{H} \to 0 \ldots\ldots\ldots\ldots\ldots$$

An examination of Eqs. 6 and 7 indicates that Eq. 22 is simply

$$V_y^2 \cong 2\left(\frac{d^2y}{dt^2}\right) y, \quad t \to 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

he expression for velocity when the acceleration is constant. This ap-
imation may be used to start the integration, since for constant accelera-

$$t \cong \left[\left(\frac{2\,y}{\frac{d^2y}{dt^2}}\right)\right]^{1/2} \qquad (24)$$

small values of t

DISCHARGE THROUGH A WEIR ANNULUS

t is fairly apparent from the foregoing material that to obtain the rapid nage required for the rapid shutting down of a reactor it is necessary to

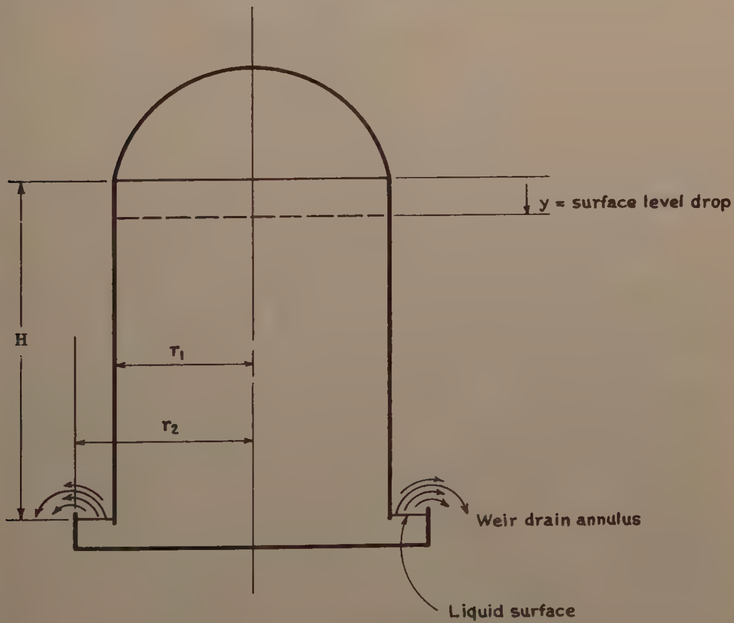

FIG. 7.—CONFIGURATION OF WEIR DRAIN SYSTEM

a large discharge area. Rather than use a very large pipe or group of s, it is expedient to use an annular weir discharge as shown in Fig. 7. In arrangement the length of the drain pipe $L = L_1 + L_2$ is assumed very ll in comparison to $(A_2/A_1)H$. The solution of Eq. 5, disregarding pres- zation and friction, is

$$\frac{V^2_y}{2\,g\,H} = \frac{R^2}{1-2\,R^2}\left[1-\frac{y}{H}-\left(1-\frac{y}{H}\right)^{(1/R^2)-1}\right] \qquad (25)$$

25 is plotted in Fig. 8 for ratios of drain pipe area to tank area of 1/2, 1/8. The corresponding terminal-flow curves are also shown. Note that

now, in the absence of any length of drain pipe, terminal flow is a better ap- proximation of the true flow, and for a practical application, would give good approximation of the time required to completely drain a tank. It i however, not satisfactory when the interest is primarily in the starting flo which it does not approximate very well.

Eq. 25 can be integrated to give the drain time explicitly. When $R \ll 1/$ the drain time is

$$T = \left(\frac{H}{2\,g}\;\frac{R^2}{1-2\,R^2}\right)^{1/2} \frac{\Gamma\left(\frac{R^2}{2-4\,R^4}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1-R^2}{2}\right)} \quad \ldots\ldots\ldots\ldots (26$$

in which Γ is the gamma function. When $R \gg 1/\sqrt{2}$

$$T = \left(\frac{H}{2\,g}\;\frac{R^2}{2\,R^2-1}\right)^{1/2} \frac{\left(\frac{1-3\,R^2}{2-4\,R^2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{2-5\,R^2}{2-4\,R^2}\right)} \quad \ldots\ldots\ldots\ldots\ldots (26$$

For $R = 1/\sqrt{2}$, Eq. 25 becomes

$$\frac{V_y^2}{2gH} = \left(1-\frac{y}{H}\right)\ln\left(\frac{1}{1-\frac{y}{H}}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2$$

As $1/\sqrt{2}$ is a very large area ratio, it is well represented in the early sta flow by free-fall. This is shown in Fig. 9.

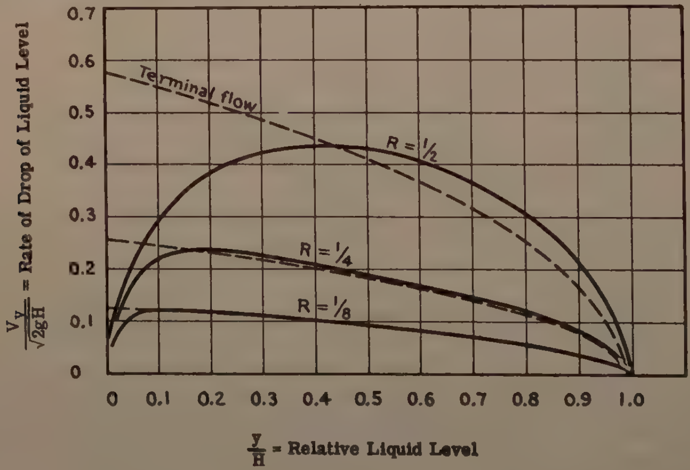

FIG. 8.—VARIATION OF FREE SURFACE VELOCITY WITH LIQUID LEVEL IN TANK. $L \ll (A_2/A_1)H$; $n=0$; $F=0$.

The time required for the complete draining of the tank when $R = 1/\sqrt{2}$ obtained by integrating Eq. 27 or from Eqs. 26. It is

$$T = \left(\frac{\pi H}{g}\right)^{1/2} \quad \text{(28)}$$

10 shows the elevation of the liquid surface at any time. It is plotted for $1/\sqrt{2}$, for free-fall, and for (improperly assumed) terminal flow. Note until about three tenths of the tank has been drained the time-displacement tionship is practically that of free-fall. The divergence becomes more ificant as the draining of the tank nears completion. To complete the ning, 25% more time is required than the time it takes for a body to fall ly through this distance. Terminal flow is not at all representative of the state of affairs. The fact that the free-fall and terminal flow curve meet /H = 1 is fortuitous.

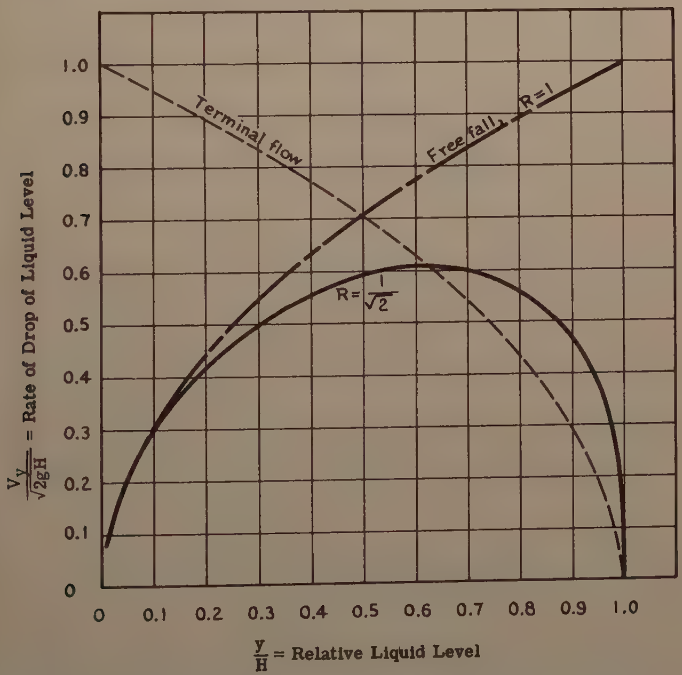

FIG. 9.—VARIATION OF FREE SURFACE VELOCITY WITH LIQUID LEVEL IN TANK. $L << (A_2/A_1)H$; $n=0$; $F=0$; $R=1/\sqrt{2}$.

CAVITATION

t the uppermost point of the drain pipe, near the junction with the tank, pressure in the liquid may be quite low at some stage of the flow. If the sure at this point is less than the vapor pressure of the liquid, then flow ration or cavitation may occur. The pressure at this point is obtained Eq. 3. It is

$$\frac{P_2}{w} = \frac{P_{02}}{w} - L_1 + \frac{L}{g}\frac{d^2x}{dt^2} + \frac{F}{2\,g}\left(\frac{dx}{dt}\right)^2 \quad \text{(29)}$$

In Eq. 29, P_{02} is atmospheric pressure and P_2 is the absolute pressure. [...] P_{cr} is the vapor pressure of the liquid then $(P_{02}/w) - (P_{cr}/w) = h_{cr}$ wou[...]

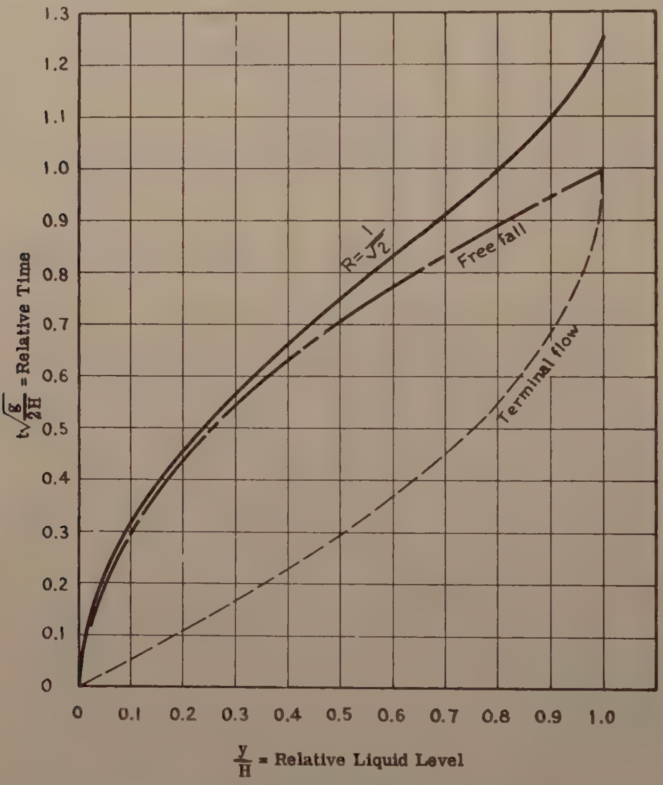

FIG. 10.—CHANGE IN LIQUID LEVEL WITH TIME. $L << (A_2/A_1)H$; $n=0$; $F=0$; $R=1/\sqrt{2}$.

represent the height of a column of liquid that can be supported when separa[...]tion is about to occur. Separation will not occur as long as $(P_{cr}/w) < (P_2/$[...] or

$$\frac{P_{cr}}{w} < \frac{P_{02}}{w} - L_1 + \frac{L}{g}\frac{d^2x}{dt^2} + \frac{F}{2\,g}\left(\frac{dx}{dt}\right)^2 \quad \ldots\ldots\ldots\ldots \quad (3$$

and separation will occur when

$$L_1 - \frac{L}{g}\frac{d^2x}{dt^2} - \frac{F}{2\,g}\left(\frac{dx}{dt}\right)^2 > h_{cr} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3$$

Thus when the fluid is stationary and not accelerating $(d^2x/dt^2 = dx/dt =$ [...] separation occurs when the vertical leg L_1 exceeds h_{cr}. At the start of flo[...]

n dx/dt = 0, in order for separation to occur, it would be necessary that

$$\frac{L}{g}\,\frac{d^2x}{dt^2} < L_1 - h_{cr} \quad \text{(32)}$$

it any given time or at any liquid-level displacement there will be an as-ated acceleration and velocity. When the acceleration and velocity fail to sfy the inequality, Eq. 31, it will demonstrate that no separation has taken e. When separation takes place the draining of the tank is no longer erned by Eq. 5.

Before the pressure at any point in the pipe reaches the absolute vapor ssure of the fluid and produces flow separation, it will pass through a pres-e range less than atmospheric. There would then be a tendency for air to k up through the pipes. However, as in the case of a siphon, as long as e is a substantial flow velocity in the drain pipe the air will not move nter the flow and the pipe will flow full.

SUMMARY

The major findings of the foregoing study of tank draining are the follow-

. When the ratio of drain pipe area to tank cross-sectional area is small the length of drain pipe is short, then the conventional manner of comput-the rate of draining, which disregards the inertia of the fluid, may be used btain reasonably accurate results.

. When the area of the drain pipe is a large fraction of the tank area, the of draining is given by Eq. 17. The neglect of the fluid inertia, in this e, leads to incorrect results.

. When the length of the drain pipe is a large fraction of, or greater than initial height of fluid in the tank, it is necessary to use Eq. 17 to compute rate of draining, and neglect of the inertia of the fluid, here again, leads ncorrect results.

. Since either large area ratios or long drain pipes have a strong influ-e on the flow, a combination of moderate area ratio and drain pipe length requires Eq. 17 for the proper computation of the rate of draining.

. As one may expect, the effect of drain-pipe friction is to decrease the of draining while the effect of tank pressurization is to increase the rate raining. For expediency the form of the friction term was taken as the dy state form, although the flow is actually non-steady.

APPENDIX.—DERIVATION OF MOMENTUM EQUATION BY ENERGY CONSIDERATIONS

The kinetic energy of the liquid in the tank is

$$\text{K.E.} = \frac{w\,A_1\,(H-y)\,\dot{y}^2}{2\,g} \quad \text{(33)}$$

and in the drain pipe it is

$$K.E. = \frac{w\, A_2 (L_1+L_2)\dot{x}^2}{2\, g} = \frac{w\, A_2\, L\, \dot{x}^2}{2\, g} \quad \text{(34}$$

The potential energy, using x = 0 as a datum is

$$P.E. = \frac{w}{2} A_1 (H-y)^2 \quad \text{(35}$$

for the tank liquid and

$$P.E. = -\frac{w}{2} A_2\, L_1^2 - w\, A_2\, L_1\, L_2 \quad \text{(36}$$

for the drain pipe. The rate at which kinetic and potential energy is bein lost from the system, in the form of draining liquid is

$$\dot{x}\left(-\frac{w\, A_2\, \dot{x}^2}{2\, g} + \frac{w}{g} A_2\, L_1\right)$$

and is equal to the rate of change with time of the kinetic and potential energ as given by Eqs. 33 through 36:

$$\frac{d}{dt}(K.E.+P.E.) = \frac{d}{dt}\left[\frac{w}{2\, g} A_1 \left(H-\frac{A_2}{A_1}x\right)\left(\frac{A_2}{A_1}\right)^2 \dot{x}^2\right.$$

$$\left. + \frac{w\, A_2}{2\, g} L\, \dot{x}^2 + w\, A_1 \left(H-\frac{A_2}{A_1}x\right)^2 - \frac{w\, A_2\, L_1^2}{2} - w\, A_2\, L_1\, L_2\right]$$

$$= -\frac{w\, A_2\, \dot{x}^3}{2\, g} + w\, A_2\, L_1\, \dot{x} \quad \text{(3}$$

Eq. 37 yields

$$\left[L + \frac{A_2}{A_1} H - \left(\frac{A_2}{A_1}\right)^2 x\right]\ddot{x} + \frac{1}{2}\left[1-\left(\frac{A_2}{A_1}\right)^2\right]\dot{x}^2 - g\left[H+L_1-\frac{A_2}{A_1}x\right] = 0. \quad \text{(3}$$

Eq. 38 is identical to Eq. 5 with n H = F = 0, which was derived from momen um considerations.

ACKNOWLEDGMENTS

J. E. Wilkins, Jr., of Nuclear Development Corp. of America deriv the material contained in the Appendix.

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CONTENTS

DISCUSSION

PROBLEMS CONCERNING USE OF LOW HEAD RADIAL GATES[a]

Discussion by Robert G. Cox and Arthur Toch
Closure by Thomas J. Rhone

OBERT G. COX,[1] M. ASCE.—The author is to be complimented for his llent paper covering the problems involved in the design and operation of ead radial gates. The writer is particularly interested in the discussion scharge characteristics.

ne fourth method presented by Mr. Rhone appears to be identical to the edure, used by the U.S. Army Engineer Waterways Experiment Station, to lop a discharge coefficient chart for radial gates on curved spillway ts. This chart was prepared for and included in "Hydraulic Design Cri-" sponsored by the Office, Chf. of Engineers, Dept. of the Army.[2] The chart (Fig. 1 herewith) and Fig. 6 of the author's paper are quite simi-except for omission of the data points on Fig. 6.

ne boundary geometry of flow under a radial gate on a curved crest is lex. To simplify discharge computations many engineers have used gate ngs and heads based on the gate seat and crest elevations. Furthermore, ost instances, the effects of the direction of the approaching streamlines been ignored. Consequently, the results are often misleading when ap- to other structures of dissimilar geometry.

ne WES method involves the net gate opening between the gate lip and the ed crest. It also includes the head to the center of the gate opening as-ng theoretical orifice discharge and atmospheric pressure under the is- jet. Discharge coefficients were computed and plotted as a function of ngle formed by the tangents to the gate-lip and to the curved crest at the ate opening. It is believed that this angle controls the direction of the mlines in the immediate vicinity of the gate opening. The results should fore be applicable to geometrically similar structures.

ne WES chart was developed using model and prototype data. The data s shown are for spillway crests shaped to the form of the lower nappe of e jet over a sharp crested weir. The effects of gate opening-head ratios e discharge coefficients were studied. No well defined families of curves be drawn. Therefore, an average curve was accepted for design pur-s.

r. Rhone's Table 1 indicates unusually good agreement between the meas- prototype and model discharges. The tabulated discharge computed by

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'ebruary, 1959, by Thomas J. Rhone.
Chf., Analysis Sect., Hydr. Analysis Branch, U.S. Army Engr. Waterways Experi- Sta., Vicksburg, Miss.
'Analysis of Hydraulic Data and Development of Design Criteria," Civil Works In-gation 804.

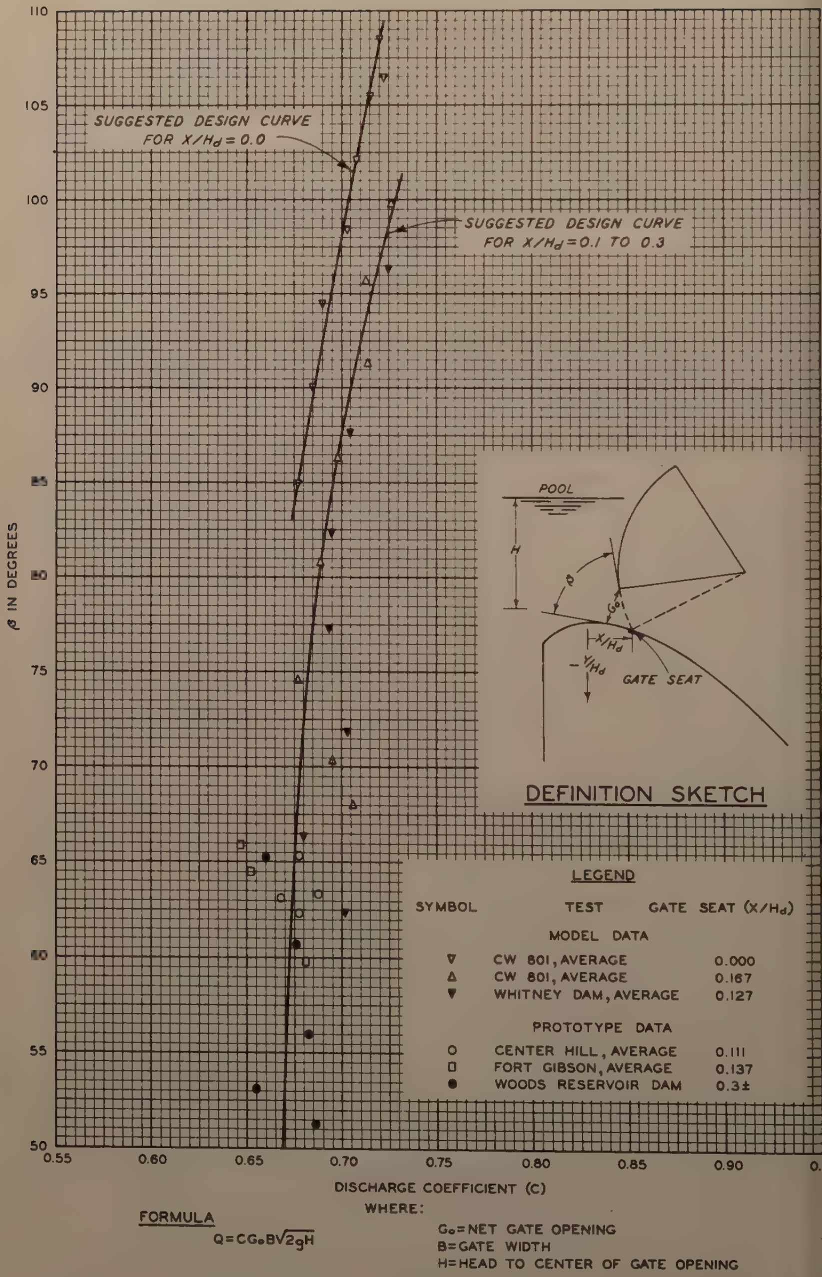

FIG. 1.—DISCHARGE COEFFICIENTS FOR RADIAL GATES.

ourth method for the 3-ft gate opening is 4,880 cfs which is about 11%
ter than the measured prototype discharge of 4,400 cfs. The writer, in an
npt to discover the reason for this difference, computed a discharge of
cfs. The latter figure is within 2% of the measured discharge. A net
opening of 2.90 ft, an effective head of 30.75 ft, and a β of 73.1° were de-
ined graphically from available data.[3] A discharge coefficient of 0.681
Fig. 1 was used in the computation. The writer would be interested in
ng comparative figures used by the author for his computation. Similar
outations for the 6-ft and 10-ft gate openings were in close agreement
discharges computed by Mr. Rhone which are about 5% greater than the
sured prototype discharges. This difference may result from the fact that
on Ferry crest-shape, downstream from the gate seat, is considerably
er than the WES shape.

the study of model data, it was noted that the discharge coefficients in-
sed rapidly for gate openings less than 0.1 ft model. For similar proto-
gate openings the coefficients were essentially constant or showed a
t decrease. This difference between model and prototype data is attrib-
to inherent difficulties in accurately measuring small gate openings and
s in models. Therefore, only model data with gate openings greater than
t were used in preparation of Fig. 1.

he writer concurs with Mr. Rhone's comments concerning the need for
ting the effects of such factors as approach depth, pier shape, gate radi-
runnion position, etc. However, it would also be necessary to determine
ffects of numerous combinations of these variables. A less elaborate ap-
ch for early usable results would be to standardize some of the geometri-
actors in terms of the design head.

RTHUR TOCH,[4] M. ASCE.—As noted previously,[5] it is believed that a
ral mathematical analysis for the radial gate in a flat-bottom, rectangu-
ection is not practical. Furthermore, it is felt that continued use of
on's article (3) as a reference in the literature on Tainter gates is re-
able. Horton not only chose to equate a contraction coefficient to a dis-
ge coefficient but did so for gate shapes of quite dissimilar geometries.

f the four methods presented by Mr. Rhone, only the last appears to dif-
rom the others. The first two formulas are evidently the same, as the
ant value $(2/3)\sqrt{2g}$ can be incorporated in the discharge coefficient
out any loss of generality for engineering applications. Both C's must be
cted, therefore, to vary in like manner, if the geometrical characteris-
of the efflux boundaries are comparable.

ne third formula differs from the first two only in the inclusion of the
$1/\sin\theta$. It seems questionable to assume that inclusion of the lip angle
s manner is of significance. Also, it is apparent that Fig. 5 merely re-
the cosecant and cosine of any given angle.

ecause of the algebraic difference between the first three formulas and
ourth, the coefficient of discharge, for the latter, will have a quite dif-
t numerical magnitude; it must still be dependent, however, on the same

"Model-Prototype Conformance of Radial Gate Discharge Capacity, Canyon Ferry
Montana," U.S.B.R., Hydr. Lab. Report No. Hyd 433, March 4, 1957.
Research Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.
"Discharge Characteristics of a Tainter Gate on a Spillway," by Z. M. Glowiak,
Thesis, State Univ. of Iowa, 1955.

variables or parameters. In fact, the importance of this aspect of the proble can hardly be overemphasized: The coefficient of discharge must be expect to be a function of the geometric configuration of gate and dam crest. In th respect, then, none of the methods presented by the author appears likely shed much light on the conditions of efflux or to increase the precision discharge prediction.

In connection with Mr. Rhone's request for further information regardi tests on radical gates, a study[5] made at the Iowa Institute of Hydraulic R search should be cited. Although this work did not include as many variabl as the author thinks necessary or desirable, Z. M. Glowiak did discuss a arrange the results he obtained in a rational manner. Another report[6] co tains fairly complete data on flow conditions and pressure distributions o tained on a model of a radial gate on a relatively high dam.

THOMAS J. RHONE,[7] M. ASCE.—Messrs. Pariset and Michel have broug out some interesting facts in their discussion, and their extension of the c efficient-of-discharge curves is, indeed, a valuable addition. None of t crest profiles used in obtaining the curves presented in the writer's pap was as thin as the Edgard de Sonza Dam crest profile; in fact, the highe free-flow coefficient was only 3.84. The larger values of the controlled fl coefficients, and the fact that for a given H the coefficient becomes essentia ly constant when θ is greater than 70°, rather than increasing, as shown Fig. 6, indicates that other variables should possibly be considered in analysis. The use of the cavitation coefficient presented is a very good mea ure for potential cavitation damage. A similar equation[8] has been used as means of determining cavitation potential in conduits and entrances, and the case of gate slots, has been proved with model prototype comparisons The writer assumes that the statement made by Messrs. Pariset and Mich "To determine the maximum allowable pressure on the profile of a spillway is in error, and should read either "minimum allowable pressure," or "ma imum allowable subatmospheric pressure," as stated later.

Mr. Bowman's awareness of the problems presented in the paper and l subsequent comprehensive discussion are very gratifying. The increasi number of papers that are appearing in technical publications seem to in cate a favorable trend in "sharing the knowledge gained from basic researcl It is hoped that this policy will continue, and will be as well received as this instance.

Mr. Bowman's discussion of the various discharge quantity equations p vides a clear, concise analysis of the advantages and disadvantages of ea He also provides a guide for the selection of the best combination of equatic to use under various conditions of gate design, gate opening, and head abo gate seal. His own method of obtaining an approximate rating curve is an e cellent "short-cut" system, and is probably as accurate as any of the me ods provided by the writer. The corrections for velocity of approach and c

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6 "Deflusso Sopra Dighe Tracimate, Somontate de Paratoie a Settore," (Flow o Dams Surmounted by Sector Gates), by V. Calderini, L'Energia Elletrica, Milan, XV, 1938.

7 Hydr. Research Engr., Div. of Engrg. Lab., Bur. of Reclamation, Denver, Colo.

8 "Engineering Hydraulics," edited by Hunter Rouse, John Wiley and Sons, p. 30.

9 "Hydraulic Characteristics of Gate Slots," by J. W. Ball, Journal of the Hyd Div., Proceedings, ASCE, Volume 85, No. HY 10, October, 1953, pp. 81-114.

tions, etc., are, for the most part, a matter of judgment, and will vary rding to the practice of each design office.

he datum profiles used in the series of tests described in the paper were d on the hydraulic model investigations of the Hoover Dam spillway.[10]

Ir. Bowman's discussion on gate seals is very informative. His observa-s on prototype installations broaden the scope of the information provided e paper, which, as he stated, was principally derived from Bureau of amation practices. The writer cannot answer the question as to how the al negotiates the bottom corners of a radial gate. In most of the struc-s investigated, the side seals were of the molded angle type, and the bot-seals were usually of the rectangular type, with a tightly fitted butt joint e corners.

n his discussion, Mr. Cox has brought forth an unfortunate omission; ely, that the writer did not identify the source of Fig. 6. The curves were ined from the Waterways Experiment Station chart, as he stated. The harge quantity that he questioned was given as 4,880 cfs in the table, but ld have been 4,580 cfs, which is within 4% of the measured quantity. The er's figures in arriving at this quantity were as follows:

Net gate opening	2.96 ft
Effective head	30.77 ft
β	72 7°
Discharge coefficient	0.681

he small discrepancies between Mr. Cox's values and the writer's can ly be attributed to different interpretations of the graphical portions of solution, and either could be accepted. Mr. Cox's explanation for the er values for the computed discharges at bigger gate openings is entirely ible. A steeper crest would permit higher flow quantities, other conditions g equal, than a flat crest. This is particularly true for the larger gate ings, as shown by Messrs. Pariset and Michel, and Bowman. The writer es with Mr. Cox's statement concerning the standardization of some of eometrical design. The use of the design head as a basis for obtaining ensionless ratios is successfully used in other phases of spillway design, should be equally satisfactory in radial gate design.

Ir. Toch implies that the discharge formulas presented in the paper are recommended by the writer; this was not the writer's intention. The four hods presented in the paper are in more or less common use, and the pur-in analyzing and discussing them was to show that each method is inade-e to some degree. The similarity between the first two formulas is in the of the equation only. It should be obvious that the coefficient-of-dis-ge values in the first equation are average values obtained from numer-model studies, irrespective of gate geometry. In the second equation, the ficients are related to the geometrical configuration of the gate. Appar-, the originator of the third method believed that the use of any gate on a t would affect the coefficient of discharge to the extent shown in the lower h on Fig. 5, and that the gate geometry would further modify the coeffi-t in the manner shown in the upper graph. The writer cannot agree that pper graph "merely relates the cosecant and cosine of any given angle,"

) "Studies of Crests for Overfall Dams," Boulder Canyon Projects, Final Reports, VI—Hydraulic Investigations, Bulletin 3.

as stated by Mr. Toch. It seems equally apparent that the graph presents simplified method for obtaining the value of $1/\sin\theta$ by using the read available values of the gate radius, trunnion height, and gate opening abo crest. Mr. Toch is correct in believing that the coefficient of discharge also a function of the crest shape. Previous discussions have vividly demo strated this to be true, and it should be an important consideration in ge eralizing a radial gate discharge equation. The two references listed by M Toch are appreciated. The thesis by Mr. Glowiak has recently been review by the writer, and it is a competent piece of research work that adds co siderably to the knowledge of radial gate discharge characteristics.

DISCUSSION OF RELATIONSHIPS BETWEEN PIPE RESISTANCE FORMULAS[a]

Closure by Walter L. Moore

'ALTER L. MOORE,[1] M. ASCE.—Several of the discussors, recognizing inconvenience of the Colebrook-White relations for solving resistance blems, presented different approaches to aid in the achievement of a more venient method. Emphasis was given to the limitations of the Colebrook-te transition function for unusual roughness forms and thus contributed to ore comprehensive presentation of the resistance problem. Some of the ements in the discussions indicated a misunderstanding of the paper and e will be cleared up to the extent possible.

Ir. Ackers' discussion was particularly interesting in that he described a erent attempt to present the Colebrook-White relations in a more usable n than previously available. The first of the two papers to which he refers s an excellent comprehensive review of the pipe-resistance problem. In ence, the approach described by Mr. Ackers consists of plotting the Cole-ok-White relations in terms of dimensionless velocity, size, and slope imeters. It is of interest to see how the parameters he used can be formed the combination of the more conventional dimensionless parameters of tion factor f, Reynolds Number N_R, and relative roughness e/D. Ackers' ensionless parameters are listed below, together with their definition from paper and their equivalent in terms of conventional parameters:

$$V = V\ e/\nu = N_R \frac{e}{D}$$

$$R = \frac{1}{4}\frac{D}{e} = \frac{1}{4} \Big/ \frac{e}{D}$$

$$S = \frac{2\ g\ S\ e^3}{\nu^2} = f\ N_R^2 \left(\frac{e}{D}\right)^3$$

ckers' Eq. 1 the k is apparently equivalent to e as used in the rest of the er.

'he general resistance diagram, plotted in terms of the foregoing para-ers, represents a logical arrangement and is based on sound dimensional soning. It has certain advantages which are readily apparent for problems lving a simple single pipe. A direct solution may be obtained for either velocity, the diameter, or the head loss, if the other necessary quantities

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[a] March, 1959, by Walter L. Moore.
[1] Prof. of Civ. Engrg., The Univ. of Texas, Austin, Tex.

are known. It is, thus, definitely more convenient than the Moody diagram f problems in which the diameter, the velocity, or the discharge are unknown.

The new diagram proposed by Mr. Ackers is not an improvement in all r spects however. Since each of the parameters V, R, and S varies over a wi range of practical values all of the scales must be rather compressed, th impairing the precision with which values can be read from the charts. course, large charts can be made to improve the precision (see charts in t pouch of Ackers' reference 7) but these become rather awkward to use a are not likely to be widely available. On the other hand, the pipe-resistan factor f varies over a rather narrow range of numerical values, and in t Moody diagram the scale for f can be chosen large enough to read with t necessary precision.

The physical significance of the variables is not as readily apparent in t Ackers' diagram as it is in the Moody diagram. The interrelationship of t velocity, size, and head loss (or slope) is apparent for the proposed diagra but this is already apparent from the general form of the various resistan relations. The significance of changes in the roughness or the fluid viscos are not readily apparent from the new diagram, because these variables o cur in more than one of the dimensionless parameters. The Moody diagra on the other hand, clarifies the significance of these variables and brings c more clearly the meaning of the "smooth pipe" and "rough pipe" limits.

For complex pipe systems, including transition losses or pipes in serie the writer can see little advantage in the Ackers diagram as compared w the Moody diagram. If the rate of discharge or the velocity is known, eith diagram may be easily used. If the discharge is not known a trial solution w be required with either of the diagrams.

It is for these complex pipe systems, as well as for pipe networks, that t writer has found the exponential relations so useful. The writer believes it more rational to examine the range of Reynolds Number and relative roug ness likely to be encountered and to develop a suitable exponential resistan equation from the modified Moody diagram, according to the methods he h proposed, than to arbitrarily "select" an exponential formula without due co sideration for the range of parameters involved in the particular problem.

Mr. Szesztay briefly reviews his work applying the Colebrook-Wh formula to a graphic solution of channel flow. Essentially his method consis of specifying the shape parameters of the cross section in terms of the wat depth h, thus making the area and hydraulic radius a function of water dept only. For a given sand-grain roughness k the discharge may then be e pressed as a function of water depth h and slope by means of the Cru modification of the Colebrook-White formula, as given in Szesztay's Eqs and 4. The chart presented in Fig. 3 permits a ready solution of these re tions. One could easily alter the relations given by Mr. Szesztay for the cha nel shape as a function of water depth, if he wished to use different crite for determining the channel shape.

Although Fig. 3 is clear for the assumed k-value, it is not clear to writer how the corresponding value of n = 0.025 can be stated. It would se that the value of n corresponding to a given roughness value k would be a fu tion of the hydraulic radius and, hence, of the water depth h. Mr. Szeszt states that statistical studies of n and k apparently fail to reveal any c sistent effect of the value of the hydraulic radius R. It would seem that t result would be worthy of additional study.

In Fig. 3, the key for the slope S is listed as centimeters over centimeters ereas it appears that it should be centimeters over kilometers.

The writer was pleased to note the similarity of viewpoint presented by Miles. His extension of the concepts to a tabular comparison of e/D and en and Williams' C-values was particularly valuable. The results that the en-Williams C approaches 160 as the relative roughness approaches 0 was eresting. The reasonableness of this value is indicated by the C of 155 for bituminous enameled-lined pipe of 16 in. or larger given in the 5th edition Babbitt and Doland's "Water Supply Engineering."

The writer is grateful to Mr. Bilonok and Mr. McPherson for correcting a merical error in the first example of the appendix. The figures shown give alue for h_f of 4.88 ft rather than 5.84 ft, as printed in the paper. The cor-t figure results in a value of $K_o = 0.455$ thus giving a resistance equation this example of:

$$h_f = 0.455\ Q^{1.88}$$

s numerical error also led to an incorrect value for the length of the 12-in. el pipe appearing in Table 2. In order that the equation for the 12-in. steel e, $h_{f_{12}} = 1.99\ Q^{1.88}$ be correct, the length of the pipe should be 4,370 ft in ce of the 3,660 ft shown in Table 2.

Mr. Bilonok also recognized that exponential formulas are in common use practicing engineers because of their convenience, as compared with the eral resistance diagram and the Darcy-Weisbach formula.

n discussing the effect of aging on pipe resistance, Mr. Bilonok states that ghness factor e, is a "complete measure of the size, shape, and distribu- of individual roughness elements." Although the introduction of roughness ameter e indicates a great improvement over other methods of charac-zing surface roughness, the writer agrees with Mr. McPherson and Mr. ers, that shape and distribution of individual roughness elements are not essarily characterized by the value of e. For the case of statistically dis-uted roughness, the single parameter e appears to be a satisfactory char-erization of the entire roughness. It has been shown that in the case of ex-me variations of roughness form, such as grooved and corrugated pipes, e other geometric parameters are important besides a measure of the ghness heights.

The writer is puzzled by Mr. Bilonok's question regarding the finding of exponent in example 2 where m = 1.95, which he says, "is not in agree-t with f and e/D." In the second example in the appendix if the values of and f, or e/D listed are used to enter Fig. 1 of the paper, the value of m be found to lie between 1.95 and 1.96.

The writer is indebted to Mr. McPherson for a very painstaking and com-hensive discussion which stressed some points which probably deserved e emphasis. Also, Mr. McPherson apparently misunderstood a number of ements in the paper.

t is obviously true that no system for approximating a complex relation- can be more precise than the relationship being approximated. Thus the posed exponential equations are certainly no more precise (but fortunately slightly less precise within the limitations imposed) than the Colebrook-e relationship, from which they were derived. Mr. McPherson states that

"the preciseness of the general resistance diagram implied by the 'exponential equations' of the author does not appear to be justified." It is a misunderstanding of the original paper to consider that the exponential equations imply about the preciseness of the general resistance diagram. They are merely an attempt to put the information in the general resistance diagram in a more convenient form for a number of types of resistance problems.

Mr. McPherson is concerned because the numerical values in Table 2 pass beyond the realm of water-works design practicality.

The table was designed merely as a means of illustrating the effect of a difference in the exponent m on the concept of pipe equivalents. Of course, the values extend beyond the range of water-works practice. It was not the writer's intention that the concept be limited only to water-works practice.

Mr. McPherson questions the statement that the exponential equation will agree with the general resistance diagram within 5% for a 20-fold variation in Q. This statement is significant, not because one expects to analyze a water distribution system over a 20-fold variation in Q, but in that it indicates that a satisfactory exponential equation may be developed even when the initially expected value of Q is poorly chosen. It must, of course, be realized that for the exponential approximation to apply for a full 20-fold variation, it must be developed about a central point in the range rather than about one extreme end of the range. Thus, in the first example in the appendix, which gives a head loss of 4.88 ft at a Q of 3.53 cfs, the comparison should be made at

$$Q = \frac{3.53}{\sqrt{20}} = 0.79 \text{ cfs}$$

and at $Q = 3.53\sqrt{20} = 15.8$ cfs.

From the equation $h_f = 0.455\, Q^{1.88}$

$$h_f = 0.290 \text{ ft at } Q = 0.79 \text{ cfs}$$

and $h_f = 82$ ft at $Q = 15.8$ cfs.

From the general resistance diagram

$$h_f = 0.299 \text{ ft at } Q = 0.79 \text{ cfs } (f = 0.019)$$

and $h_f = 85$ ft at $Q = 15.8$ cfs. $(f = 0.0135)$

The percentage differences are as follows:

at $Q = 0.79$ cfs; $(0.292/0.299 - 1) = -2.2\%$

at $Q = 15.8$ cfs; $(82/85 - 1) = -3.5\%$

The writer agrees with Mr. McPherson that in designs of water-works networks a constant value of the exponent m may normally be used. The writer believes, however, that an exponent chosen from the modified Moody diagram is more rational and will also give better results than arbitrarily using the 1.85 exponent from the Hazen-Williams formula. Mr. McPherson's discussion does not seem to recognize that network problems also exist in fields other than water-works practice. In some instances it may even be desirable to use different exponents m for different branches of the system. Contrary to Mr. McPherson's statement, digital computers are not necessarily restricted to using exponent m.[2,3]

2 "More on Pipe Line Computers," by Robert L. McIntire. Oil and Gas Journal, Vol. 57, No. 26, June 22, 1959.

Mr. McPherson has apparently not understood the writer's use of the term, "exponential resistance equations," which was defined in the paragraph immediately following Eq. 2. Thus, any equation of the form $h_f = K\,Q^m$ (which includes the Hazen-Williams formula as a special case) is considered an exponential formula. Mr. McPherson must have misunderstood the term in order to make the statement that he was "forced to reject as obviously unrealistic (and clearly not proven) the statement by the author: 'although the methods for analysis of networks will not be considered here, it is pertinent to note that exponential resistance relations will be most convenient for use in this type of analysis.'" Mr. McPherson goes on to propose the use of the Hazen-Williams formula for network analysis which is really a confirmation of the writer's statement although restricting it to the special case of an exponent $m = 1.85$.

Again Mr. McPherson apparently misunderstood the statement that, "thus the Hazen-Williams formula with its exponent of 1.85 corresponds to a combination of relative roughness and Reynolds number lying along the line, $m = 1.85$ on the modified Moody diagram." The discussion preceding this statement clearly deals with the effect of m on the change of head loss with change in Q. The statement should thus be taken to indicate that only along the line $m = 1.85$ will the Hazen-Williams equation have the correct exponent. Obviously the whole field of friction factors and Reynolds numbers may be covered by lines corresponding to an exponent $m = 1.85$. This is what is done in the chart presented by Mr. McPherson as Fig. D-2. The important point is that the change in f with Reynolds number occurs at the same rate for both the Hazen-Williams equation and the general resistance diagram, only along the line corresponding to $m = 1.85$. This may be clearly shown by plotting coordinates of the line $m = 1.85$ from the modified Moody diagram onto Fig. D-2. It will be seen that only along this line are the solid lines representing the Hazen-Williams equation approximately parallel to the dashed lines representing constant relative roughness on the general resistance diagram. To the right of the chart the Hazen-Williams equation would indicate a continuing decrease of friction factor with increasing Reynolds number, whereas the rough pipe relations represented on the resistance diagram show that the friction factor should become constant. It might be argued that $C_{H.W.}$ could be decreased as Reynolds number is increased, thus forcing Hazen-Williams equation to yield a constant f at high Reynolds number. This would completely destroy the added convenience of the exponential type equation in this range. Clearly what is needed in the rough-pipe range is an equation with an exponent $m = 2$ in order to fit the conditions for a constant f-value.

Further evidence of the correctness of the original statement is given by the first equation presented by Mr. McPherson under the heading, "The Hazen-Williams Formula." This equation

$$f_{H.W.} = \frac{1060}{C_{H.W.}\ N_R^{0.15}\ D^{0.015}}$$

the same form as Eq. 5 of the paper with $m = 1.85$ because $D^{0.015}$ will be practically 1 for any reasonable value of D. Thus if C_o is set equal to $1060/C_{H.W.}$ and $m = 1.85$, Eq. 5 will be seen to correspond to the foregoing equation.

[3] "Relaxation Methods for Pipe Networks," by T. G. Chapman. Civ. Engrg. and Pub. Works Review, Vol. 51, No. 605, Nov. 1956.

The data presented by Mr. McPherson in Tables D-3 and D-4 for new linings and old linings is of interest. If the C-values given for Reynolds number and friction factor are plotted on the modified diagram, the points for the new linings, for the most part, fall between the lines of m = 1.90 and m = 1.96. For the old linings (Table D-4) the points, for the most part, lie between the lines m = 1.94 and m = 1.975. This data, also, indicates that the resistance relations for the older pipes would be better approximated by a larger value of the exponent m than for the new pipes. It is also of interest that the indicated values of m are all considerably about the 1.85 line corresponding to the Hazen-Williams equation. Thus this data also supports the writer's contention that an exponential equation with an exponent larger than 1.85, in fact one approaching 2, would be preferable for old water mains.

It is the writer's belief that the modified Moody diagram in conjunction with available data for e, or better still test results such as presented by Mr. McPherson, will enable the engineer to select the exponential equation which will be most appropriate for the type of problem with which he is dealing. The writer recognizes that there is a possibility (which at this time seems rather remote) that large changes in roughness form due to encrustation and the forming of ridges or waves might alter the form of the transition function from "smooth" to "rough" pipe conditions and thus affect the proper value of the exponent m. That the shape of the roughness should change enough to significantly affect the treatment function seems unlikely, and in the absence of reliable experimental data on this point, the modified Moody diagram appears to be a reasonable improvement over existing methods.

Additional research to explore the shape of the transition function and the geometric form of roughness for encrusted pipes would certainly be of interest. In such an investigation it may well be that the reduction in diameter will be more significant than the change in roughness since the diameter enters into the equations to the fifth power.

The work described by Dawsey attempting to relate measurements of roughness profiles to the equivalent sand grain roughness e has much appeal. To be able to run a test on a small portion of a conduit lining, or better still on a cast taken of a portion of the lining, and convert this to a hydraulic roughness parameter would be a tremendous forward step. There are many difficult problems of measurement, and meaningful analysis of the measurements, that will have to be solved before this approach is likely to yield reliable results. The magnitude of the potential gains are so great that efforts toward this goal seem well directed. The writer thanks Mr. Dawsey for noting the omission of parentheses in the Colebrook-White equation as presented in Table 1 of the paper.

The writer is grateful to all of the discussors for contributing additional information and new thoughts regarding pipe-resistance relations. The discussions have supplemented the paper by presenting other attempts to develop convenient as well as reliable methods of evaluating pipe-resistance. It is hoped that the concepts presented will lead to an improved understanding of pipe resistance relations. Time and the experience of the practicing engineer will determine which of the various approaches will prove to be most useful.

CHANNEL-SLOPE FACTOR IN FLOOD-FREQUENCY ANALYSIS[a]

Closure by Manuel A. Benson

MANUEL A. BENSON,[1] M. ASCE.—Mr. Sammons is to be commended for is thorough analysis and reanalysis of the writer's data. The original article as intended to cover only the choice of a simple, yet efficient, index of main-hannel slope. Mr. Sammons covered a wide scope, including basic data col-ection, mapping standards, flood-frequency analysis, multiple correlation, tatistics, and the use of automatic computers. Much of his discussion con-erns points which will be explained in detail in the pending final report of the ompleted project. Mr. Sammons makes some sweeping indictments of vari-us practices, to most of which the writer wishes to plead "not guilty." For xample, the writer sincerely believes he does not share that misconception the use of automatic computers, and he is certain that he does not apply ndiscriminate weighting coefficients" to variables.

Mr. Sammons puts much stress on the distribution of the variables which re used in the regression analysis. Multiple-correlation techniques, to be alid, require linear relationships between the variables. The transformation logarithms of the original variables was made on the basis of comparative garithmic and rectangular plotting between the dependent and independent ariables, and plottings of the residual errors at various stages against the dependent variables. One variable (surface-storage area in ponds) used in e final analysis was adjusted by a "rectification constant" determined graph-ally to be necessary to linearize the logarithms of the variable. Finally (as ggested by Mr. Sammons) the cumulative distributions of the variables entually used were plotted on log-probability paper and were proved to be sentially linear, within 2 standard deviations of their means. This con-rmed the validity of the logarithmic transformations.

No logical reason is apparent for Mr. Sammons' statement that the curvi-near regression constant of "a" in Fig. 3, indicate that "a curvilinear garithmic model would best fit the data." Elsewhere, Mr. Sammons refers the "concave upward bias of the constant term a on Fig. 3." A bias is much ore apt to be introduced by an a priori theory than by adherence to the data.

The distribution of most of the variables, which may influence flood peaks, sufficiently close to log-normal, thus that distribution may be assumed. In y event, their exact distribution is not critical for the purpose of relating em to flood peaks. However, the writer is not willing to assume log-normal stribution for the annual peak discharges at a station, because this is a itical assumption. The form of the distribution becomes the principal basis

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[a] April, 1959, by Manuel A. Benson.

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for determining extreme flood peaks. Statistical methods may be nonparametric as well as parametric. The writer prefers to correlate topographic and climatic variables with the "percentiles" of the flood-peak distribution rather than with its parameters (mean, standard deviation, and skew), simply because the use of parameters means making assumptions that are not necessarily or generally true and that may actually be in error.

Mr. Sammons cannot suppose that the investigation which he cites, in which precipitation proved to be more important than the drainage area, is generally applicable or reflects on the results for New England. If the analysis were made for a group of small drainage areas (necessarily limited in range of sizes) within a region of widely varying precipitation, it is to be expected that precipitation will prove more important. In fact, if all the drainage areas are the same size, drainage area would be of no importance whatsoever, at least as shown by the multiple-regression analysis.

The writer agrees with Mr. Sammons that the median values of variables might be better indices than the means. However, the U.S. Weather Bureau publishes only the means for climatic factors such as precipitation and temperature. The mean of topographic variables, such as elevation and land slope, is more simple to compute than the median.

The scales of topographic maps of New England were adequate to define profile elevations for the basins used in the analysis. Regions of flat slopes, less adequately mapped regions, or extremely small watersheds might require surveys or photogrammetry to determine the channel slope. The writer did not investigate the effect of variation in map scales because the available scales introduced no problems for the data studied.

The ultimate findings have related flood peaks of recurrence intervals ranging from 1.2 to 300 yr, to the following hydrologic variables:

1. Drainage area;
2. main-channel slope;
3. surface storage in lakes and ponds;
4. precipitation intensity;
5. mean January temperature (an index of the effect of snowmelt on peaks); and
6. orographic factor.

A report on the investigation and the findings is nearing completion.

ROLL WAVES AND SLUG FLOWS IN INCLINED OPEN CHANNELS[a]

Discussion by Tojiro Ishihara, Yuichi Iwagaki and Yoshiaki Iwasa

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TOJIRO ISHIHARA,[1] YUICHI IWAGAKI,[2] M. ASCE and YOSHIAKI WASA.[3]—The writers have reviewed this paper with interest, because we lso have made continuous investigations of the hydraulic characteristics of oll waves and other associated problems in laminar and turbulent flows from heoretical and experimental approaches during the past several years. Unortunately, however, our results are quite different from the author's study, nd we also do not follow his treatment of roll waves and slug flows from the nowledge we obtained through research in the hydraulics laboratory of Kyoto niversity.

Our research project to disclose the hydraulic characteristics of roll aves, needed by the engineering request for soil conservation, have been tudied by the writers and others since 1950. Several reports were already ublished in Japanese[4-7] and in English[8,9] after completing specific problems elated to roll waves. Through our research works, the basic concept of the reatment for the hydraulic characteristics of roll waves is essentially due to e studies of H. Thomas[10] and R.F. Dressler.[11] More precisely, the derivaon of the hydraulic characteristics of roll waves is based on the fact that

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[a] July, 1959 by Paul G. Mayer.

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[4] "Theory of the Roll-Wave Trains in Laminar Water Flow on a Steep Slope Surface," T. Ishihara, Y. Iwagaki, and Y. Iwasa. Studies on the Thin Sheet Flow, 5th Report., rans. JSCE, No. 19, April, 1954, (in Japanese).

[5] "On the Hydraulic Characteristics of the Roll-Wave Trains," by Y. Iwagaki and Iwasa. Studies on the Thin Sheet Flow, 7th Report, Journal JSCE, Vol. 40, No. 1, nuary, 1955, (in Japanese).

[6] "Mechanism of Water Erosion on Land-Surfaces and Roll-Wave Trains," by T. hihara, Y. Iwagaki and Y. Iwasa. Bulletin, Engrg. Research Inst., Kyoto Univ., Vol. 7, arch, 1955, (in Japanese).

[7] "Fundamental Studies on Land Erosions due to Rain Water Flow," by Y. Iwagaki. . Engrg. Thesis, Kyoto Univ., March, 1956, (in Japanese).

[8] "On the Rollwave-Trains Appearing in the Water Flow on a Steep Slope Surface," T. Ishihara, Y. Iwagaki and Y. Ishihara. Memoirs, Faculty of Engrg., Kyoto Univ., l. 14, No. 2, March, 1952.

[9] "The Criterion for Instability of Steady Uniform Flows in Open Channels," by Y. asa. Memoirs, Faculty of Engrg., Kyoto Univ., Vol. 16, No. 4, October, 1954.

[10] "The Propagation of Waves in Steep Prismatic Conduits," by H. Thomas. Proc. dr. Conf., Univ. of Iowa Studies in Engrg., Bulletin 20, 1940.

[11] "Mathematical Solution of the Problem of Roll-Waves in Inclined Open Channels," R.F. Dressler. Comm. on Pure and Applied Math., Vol. 2, No. 2/3, 1949.

(1) the wave velocity of roll waves is definitely constant (terminal velocity by the author), regardless of their flow regime, when the final shape in wave pattern is obtained, and (2) the flow velocity is, in all cases, less than the wave velocity of roll waves in magnitude, thus the flow regime of rear side of a roll wave is shooting, whereas that of the front side, including the wave crest, is tranquil when the train of roll waves is observed from the moving coordinate system travelling at a constant speed of wave velocity. The former condition was also verified by the author, while the latter condition of Thomas was not equivalent to the author's in all points because the author divided the flow behaviors into two different wave patterns, depending on the value of the Reynolds number.

In order to ascertain our standpoint concerning the author's derivation for the hydraulic characteristics of roll waves and slug flows, our analytical treatment on the hydraulic characteristics of roll waves and the experimental verification by the direct measurement of roll waves by means of electronic devices will be first described:

Based on the principle of a constant wave velocity indicated in the foregoing, the unsteady behaviors of roll waves can be reduced to those in steady regime by eliminating the time derivatives in the basic quasi-linear partical differential equations of hyperbolic type for unsteady flows, when viewing the wave pattern from the moving origin. The hydraulic characteristics of roll waves are then easily derived in the following.

The equation of momentum for unsteady flows in open channels with a constant inclination is

$$\frac{\partial v}{\partial t} + \alpha\ V \frac{\partial V}{\partial X} + g \cos \theta \frac{\partial D}{\partial X} + (1 - \alpha) \frac{V}{A} \frac{\partial D}{\partial t} = g \sin \theta - \frac{g\ V^2}{C^2\ R} \quad \ldots \quad (1)$$

and the equation of continuity is

$$\frac{\partial A}{\partial t} + V \frac{\partial A}{\partial X} + A \frac{\partial V}{\partial X} = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

in which the pressure distribution is assumed hydrostatic, x is the distance from the origin along the channel bed, t represents time, V is the mean velocity of flow, D denotes the depth of water, A is the flow area, R represents the hydraulic radius, C is Chézy's coefficient (assumed variable), g denotes the acceleration of gravity, θ is the inclination angle of channel bed, and α equals the momentum correction factor usually defined. Apparently, the velocity profile of laminar flow is of the parabolic type which was already derived by many hydraulic engineers as well as the author and the momentum correction factor becomes then a constant of 1.2 with a simple calculation. On the other hand, the evaluation of α in turbulent flow cannot be made in a definite number, because it is still a function of flow and channel characteristics but it is satisfactorily assumed 1.05 in engineering practice. In the derivation of Eq. 1, the curvature of a stream line is neglected, and thus the influence of surface tensions is also ignored, which is different from the author's treatment. Of course, the surface tension will have to be considered for laminar flow in very steep channels.

Taking account of the preceding principle for a constant wave velocity, the time derivatives in Eqs. 1 and 2 can be eliminated by setting $\zeta = x - V_W$ t

l the resulting equations obtained by transforming the independent variables
m x and t to ζ are

$$\frac{V}{\zeta} = \frac{g(V - V_w)(dA/dD)\{\sin\theta - (V^2/C^2 R)\}}{(dA/dD)\{(\alpha V - V_w)(V - V_w) + (1-\alpha) V V_w\} - g A \cos\theta} \quad \ldots\ldots (3)$$

l

$$\frac{D}{\zeta} = - \frac{g A\{\sin\theta - (V^2/C^2 R)\}}{(dA/dD)\{(\alpha V - V_w)(V - V_w) + (1-\alpha) V V_w\} - g A \cos\theta} \quad \ldots\ldots (4)$$

reality, the wave velocity may not be considered constant in a channel of
egular section, but the treatment of analysis will be proceeded from a
croscopic aspect for mean flows. Eliminating V from Eqs. 3 and 4, the
eadied wave pattern observed from the moving origin can be expressed by

$$\frac{D}{\zeta} = - \frac{g A(\sin\theta - \{(V_w A - K)^2/C^2 R A^2\})}{(dA/dD)\{(\alpha K^2/A^2) + (1-\alpha)V_w^2\} - g A \cos\theta} = - \frac{f_1(D)}{f_2(D)} \quad \ldots\ldots (5)$$

which K is a progressive discharge rate defined by $(V_w - V)A$ = a constant.
seen in Eq. 5, the wave pattern of a roll wave cannot be derived in a peri-
c form observed in laboratories, and thus the solution for roll waves must
obtained as a discontinuous wave pattern by combining each of water sur-
e profiles through the shock condition as Dressler did. Dressler also ob-
ned a continuous periodic solution of roll waves being in forms of the
iptic function as the second order solution of the shallow water wave theory.
e actual wave pattern, however, observed in channels is rather in a similar
m to discontinuous periodic solutions.

The basic concept of Thomas obtained by his specified experimental works
icates that the flow involves a control section in each water surface pro-
e, as the flow regime changes from tranquil to shooting, so that in Eq. 5,
h numerator and denominator become simultaneously zero and all the hy-
ulic characteristics of roll waves can be uniquely determined at this sec-
n. The resulting expressions for the mean velocity and the hydraulic radius
the water depth in specified cases at the control section are

$$\frac{V_o}{V_w} = \frac{\alpha(dA/dD)_o - \sqrt{\alpha(\alpha-1)(dA/dD)_o^2 + \{(dA/dD)_o S_o/N_F^2\}^2}}{\alpha(dA/dD)_o - (S_o/N_F^2)} \quad \ldots\ldots\ldots (6)$$

l

$$\frac{g \cos\theta}{V_w^2} = \frac{1}{N_F^2}\left[\frac{\alpha(dA/dD)_o - \sqrt{\alpha(\alpha-1)(dA/dD)_o^2 + \{(dA/dD)_o S_o/N_F^2\}^2}}{\alpha(dA/dD)_o - (S_o/N_F^2)}\right]^2 \quad \ldots (7)$$

which S is the wetted perimeter, N_F denotes the Froude number defined by
$/(g R_o \cos\theta)^{1/2}$, and the subscript o indicates the values at the control
tion which are eventually equal to the quantities in uniform flow. The pro-
ssive discharge rate can also be calculated by the use of Eqs. 6 and 7.

If the flow is laminar in a rectangular channel, the normal mean velocity
low, the normal depth of water, and the progressive discharge rate are

explicitely indicated as functions of the wave velocity and Froude number : follows:

$$\frac{V_o}{V_w} = \frac{(6/5) - \sqrt{(6/25) + (1/N_F^2)}}{(6/5) - (1/N_F^2)} \quad \ldots\ldots\ldots\ldots\ldots ($$

$$\frac{D_o\ g\ \cos\theta}{V_w^2} = \frac{1}{N_F^2}\left\{\frac{(6/5) - \sqrt{(6/25) + (1/N_F^2)}}{(6/5) - (1/N_F^2)}\right\}^2 \quad \ldots\ldots\ldots ($$

and

$$\frac{K\ g\ \cos\theta}{V_w^3} = \frac{1}{N_F^2}\left\{\frac{(6/5) - \sqrt{(6/25) + (1/N_F^2)}}{(6/5) - (1/N_F^2)}\right\}^2$$

$$1 - \left\{\frac{(6/5) - \sqrt{(6/25) + (1/N_F^2)}}{(6/5) - (1/N_F^2)}\right\} \quad \ldots\ldots\ldots\ldots (1$$

Water-surface profiles as each wave pattern of roll waves are obtained l integrating Eq. 5 upstream and downstream from the control section. A though water-surface-profile equations cannot be integrated analytically general cases, those for laminar flows and for turbulent flows, characteriz by a constant Chézy coefficient in rectangular channels, are integrable. cases of two-dimensional laminar flows, the resulting water-surface profi is indicated in an inverse functional form as follows:

$$\zeta = \frac{1}{\sin\theta}\left\{\frac{D_A^2 \cos\theta + (6/5)(K^2 D_A/g\ D_o^2) + (6/5)(K^2/g\ D_o)}{D_A - D_B}\ \log_e \frac{D - D_A}{D_o - D_A}\right.$$

$$- \frac{D_B^2 \cos\theta + (6/5)(K^2 D_B/g\ D_o^2) + (6/5)(K^2/g\ D_o)}{D_A - D_B}\ \log_e \frac{D - D_B}{D_o - D_B}$$

$$\left. + \cos\theta\ (D - D_o)\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1$$

in which the integral constant is selected so that $D = D_o$ at $\zeta = 0$, and D_A a D_B are

$$\frac{D_A}{D_o} = \sqrt{\frac{9}{20} + \sqrt{\frac{6}{25} + \frac{1}{N_F^2}}} - \frac{1}{2} \quad \ldots\ldots\ldots\ldots\ldots (1$$

$$\frac{D_B}{D_o} = - \sqrt{\frac{9}{20} + \sqrt{\frac{6}{25} + \frac{1}{N_F^2}}} + \frac{1}{2} \quad \ldots\ldots\ldots\ldots\ldots (1$$

and $D_o > D_A > 0 > D_B$.

The complete discontinuous solution of roll waves can be obtained by com-
ing each of water-surface profiles through the shock condition, as will be
scribed in the following.

The shock condition for open-channel flows involving a moving discontinu-
is expressed by the following three conditions:

Mass conservation:

$$- \rho A_b(V_w - V_b) = - \rho A_f(V_w - V_f) = M \dots\dots\dots \quad (14)$$

Momentum conservation:

$$V_w - V_f) - (V_w - V_b) = \rho g \cos \theta \{ A_f(D_f - D_{Gf}) - A_b(D_b - D_{Gb}) \} \dots \quad (15)$$

Energy dissipation rate:

$$P = \frac{dE}{dt} = \frac{M g \cos \theta}{2 A_f A_b} \{ A_b^2 (D_b - D_{Gb}) - A_b A_f (D_b + D_{Gb})$$

$$+ A_b A_f (D_f + D_{Gf}) - A_f^2 (D_f - D_{Gf}) \} \dots\dots\dots \quad (16)$$

which D_G is the distance from the channel bed to the centroid of flow area,
the subscripts of f and b indicate the values at the lowest and highest
nts of water-surface profiles. Using the expression for progressive dis-
rge rate and Eqs. 14 and 15, the relationship between the lowest and high-
depths of a wave becomes

$$K^2 \{ (1/A_b) - (1/A_f) \} = g \cos \theta \{ A_f(D_f - D_{Gf}) - A_b(D_b - D_{Gb}) \} \dots \quad (17)$$

oting the wave length of a roll wave by L, the following relationship must
t between two successive waves:

$$\zeta (D_{n+1}) = L + \zeta (D_n) \dots\dots\dots\dots\dots \quad (18)$$

at the shock front

$$\zeta_{D_n}(D_b) = \zeta_{D_{n+1}}(D_f) \dots\dots\dots\dots\dots \quad (19)$$

f the water-surface profile for a particular roll wave is obtained by inte-
ing numerically Eq. 5, the complete solution of discontinuous periodic
ern of roll waves will also be calculated graphically or numerically. In
es of two-dimensional laminar flows, the wave length of a roll wave is ex-
tly determined by

$$\frac{1}{\sin \theta} \left\{ \frac{D_A^2 \cos \theta + (6/5)(K^2 D_A / g D_0^2) + (6/5)(K^2 / g D_0)}{D_A - D_B} \log_e \frac{D_b - D_A}{D_f - D_A} \right.$$

$$- \frac{D_B^2 \cos \theta + (6/5)(K^2 D_B / g D_0^2) + (6/5)(K^2 / g D_0)}{D_A - D_B} \log_e \frac{D_b - D_B}{D_f - D_B}$$

$$\left. + \cos \theta (D_b - D_f) \right\} \dots\dots\dots\dots\dots \quad (20)$$

The criterion for roll waves to maintain their final wave pattern, as d scribed in the foregoing, will be studied next. It is of common observati that the wave profile of a roll wave is monoclinally increasing from the re side to the front side, so that the following relationship for the water dep must be obtained at the control section, when the waves are observed fro the moving origin.

$$\lim_{D=D_o} \frac{dD}{d\zeta} \geqq 0 \quad \text{. . . . . (2}$$

With the aid of Eq. 5, the above condition for laminar flows characterized the parabolic velocity profile and $\alpha = 1.2$ becomes

$$\frac{2\ m\ V_o}{V_w - V_o} \geqq 1 \quad \text{. . . . . (2}$$

or, expressing in terms of Froude number,

$$N_F^2 \geqq S_o/\left(\frac{dA}{dD}\right)_o (4\ m^2 - 0.8\ m - 0.2) \quad \text{. . . . . (2}$$

in which m is the shape parameter defined by $1 - R_o(ds/dA)_o$ and it gradua increases to unity with increase in width of a channel.

If the flow is turbulent, the criterion is also calculated in the same ma ner, and it is

$$\frac{m\ V_o}{2\ V_w}\left\{\frac{2\ R_o}{C_o}\left(\frac{dC}{dR}\right)_o + \frac{2}{m} + 1\right\} \geqq 1 \quad \text{. . . . . (}$$

for the channels in which the flow area is proportional to a power of the wa depth. To obtain the criterion in a more familiar fashion for hydraulic en neers, the velocity formula of Vedernikov's type will be introduced as

$$V_o^a = \beta\ R_o^{1+b} \sin\theta \quad \text{. . . . . (}$$

in which a, b and β are constants determined by given flow and channel ch acteristics, so that the Chézy's coefficient C becomes

$$C_o = \beta^{1/a}\ R_o^{2(1+b)-a/2a}\ (\sin\theta)^{2-a/2a} \quad \text{. . . . . (}$$

Inserting Eq. 26 into Eq. 24, the resulting condition becomes

$$V_e = \frac{(1 + b)m\,V_o)}{a(V_w - V_o)} \geqq 1 \quad \text{. . . . . (}$$

or, expressing in terms of Froude number,

$$N_F^2 \geqq S_o/\left(\frac{dA}{dD}\right)_o \left\{m^2\left(\frac{1+b}{a}\right)^2 - 2(\alpha - 1)m\left(\frac{1+b}{a}\right) - (\alpha - 1)\right\} \text{. .}$$

Eq. 27 is evidently the Vedernikov number for the initial instability of sho ing flows. The last of the writers[9] also derived the same criterion by ded

the initial condition for continuous time growth of an infinitesimally small turbance with the use of Eqs. 1 and 2. A. Craya[12] and Dressler[13] studied similar criteria, and in specified cases of particular velocity formulas ffreys[14] and Keulegan and Patterson[15] obtained the instability conditions. most significance, in deduction of the foregoing maintenance conditions for l waves being in their final wave pattern, is that two conditions for mainance of roll waves and for initial instability of shooting flows are exactly same expression of Vedernikov, regardless of essentially different apaches in deduction. The formation of roll waves, therefore, will result m the instability of open-channel flows as many hydraulic engineers deribed.

The previous explanation for the hydraulic characteristics of roll waves is ummary of our past analytical treatment developed by means of the basic cepts of Thomas and Dressler. The experimental works for the characistics of roll waves, to clarify the hydraulic behaviors of waves in definite ms and to verify the theoretical deduction of roll wave characteristics, re also paralleled by the writers in the hydraulics laboratory of Kyoto iversity. Two test flumes of a rectangular section were used for this purse, and in the earlier stage of research progress, which was mainly subt to laminar roll waves, the aluminum channel of 20 ft in length was used, ereas the experimental runs for turbulent roll waves, which are called as g flows by the author, were made in the lucite flume of 36 ft in length and variable slopes from horizontal to 30°. Continuous recordings of wave proes of roll waves and of wave velocities were made by the electronic wave corders of resistance type, and the discharge measurement was made by ume metering of water at the downstream end. The details of experimental cedures were reported in the writers' paper.[4] Comparing the theoretical ults described in the foregoing, the experimental behaviors of roll waves ained by the writers will be presented in the following. Data of the author Binnie[16] will be also supplemented.

The criterion for initial instability and formation of roll waves will be ated first. For the sake of simplicity in illustration, the flow is assumed -dimensional. In this case, Eq. 23, which indicates the criterion for lami- flows, becomes

$$N_F = 0.577 \quad \text{or} \quad \frac{1}{\tan\theta\, R_D} = 1 \qquad (29)$$

which R_D is the Reynolds number given by $V_o\, D_o/\nu$, and the same criion for turbulent flows is also in terms of Froude number,

$$N_F^2 = 1/\left\{\left(\frac{1+b}{a}\right)^2 - 2(\alpha-1)\left(\frac{1+b}{a}\right) - (\alpha-1)\right\} \qquad (30)$$

12 "The Criterion for the Possibility of Roll Wave Formation," by A. Craya. Proc. he Gravity Wave Symposium, NBS, 1951.

13 "Resistance Effects on Hydraulic Instability," by R.F. Dressler and F.V. Pohle. nm. on Pure and Applied Math., Vol. 6, No. 1, 1953.

14 "The Flow of Water in an Inclined Channel of Rectangular Section," by H. Jeffreys. l. Mag., Series 6, Vol. 49, 1925.

15 "A Criterion for Instability of Flow in Steep Channels," by G. H. Keulegan and W. Patterson. Trans. AGU, Part II, 1940.

16 "Instability in a Slightly Inclined Water Channel," by A.M. Binnie. Journal of id Mech., Vol. 5, Part 4, May, 1959.

and consequently, it depends on the velocity formulas like Chézy's or Ma ning's and the momentum correction factor. Since the Froude number is e pressed as a function of the Reynolds number, Manning's roughness coef cient, slope and kinematic viscosity, when the Manning formula is used, t instability criterion can be graphically illustrated in terms of the slope a Reynolds number as parametric expressions of the roughness and the m mentum correction factor. Fig. 1 illustrates the criterion for instability flows and formation of roll waves obtained in the preceding manner under t assumption that the kinematic viscosity is 0.01 sq cm per sec. Two zones stable and unstable flows will be apparently observed in the figure. In ope channel flow, the transition from laminar to turbulent flows takes place f Reynolds number from 500 to 1,500, as often indicated by many enginee

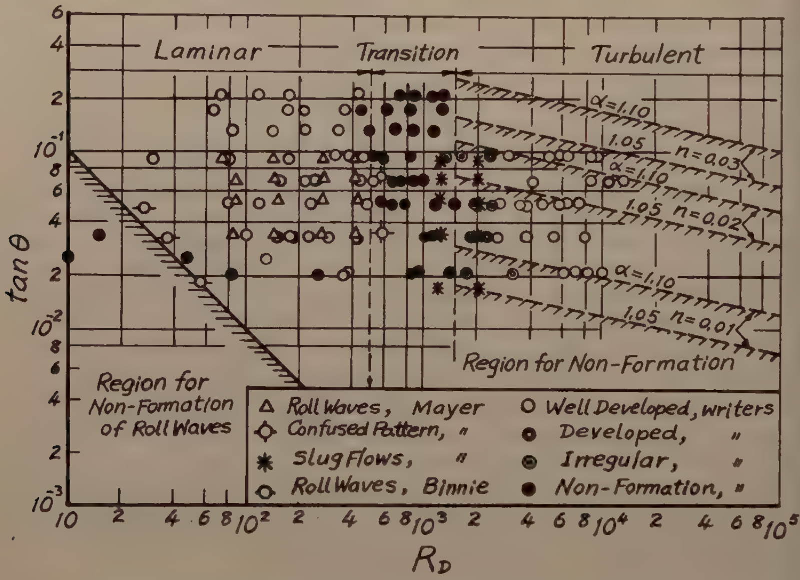


FIG. 1.—REPRESENTATION OF CRITERION FOR FORMATION OF ROLL WAVES.

The second of the writers[7] found that the Manning formula was not suitable the transition region, but (1 + b)/a was commonly very small and it beca negative in particular cases depending on the channel slope, through the e perimental research. The formation of roll waves, therefore, will not resul

Now, let us consider the behavior of roll waves in formation when the fl is carried in an open channel. For clarification in explanation, the chann slope is assumed 0.02. When the regulating valve for discharge is open from closure, the flow begins to be carried. The roll waves are not observ until the Reynolds number becomes 50, as previously indicated. With i crease in discharge, the laminar roll waves become appreciable. Once t Reynolds number increases to 500, so the transition from laminar to turb lent flows begins to take place and the roll waves disappear. This phenomen may be corresponding to that called the confused pattern by the author. W the further increase in discharge, the flow becomes completely turbulent, a

in the roll waves are predominant in forms of bore if the channel roughness is approximately less than 0.01 in Manning's roughness coefficient, ereas for rough channels in which the coefficient is larger than 0.01, the l waves can not be observed. When the discharge is still increased, the roll ves ultimately appear even in such rough channels. This fact is evidently ual to the results obtained by Thomas,[10] Dressler[11] and others.

In Fig. 1, our experimental data for roll waves are plotted, adding supplental data of the author and Binnie. It is then concluded that the foregoing atment for the formation of roll waves is verified by the experimental obvations. Evidently, the law of resistance and the momentum correction tor play a very important role in the criterion for formation of roll waves. . 2 indicates the criterion in terms of velocity formula and momentum

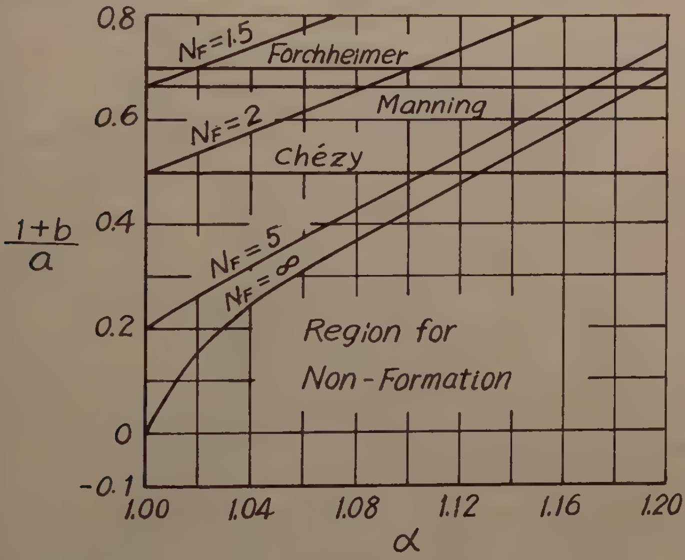


FIG. 2.–ILLUSTRATION OF CRITERION FOR FORMATION OF ROLL WAVES IN TERMS OF LAW OF RESISTANCE AND MOMENTUM CORRECTION FACTOR.

rection factor, and it also illustrates that roll waves cannot be observed in transition region from laminar to turbulent flows, as $(1 + b)/a$ is very all in this region.

In the same manner, the criterion for roll wave formation in triangular nnels will be obtained as follows.

$$N_F^2 = 4/\sin\phi \left\{ \left(\frac{1+b}{a}\right)^2 - 4(\alpha - 1)\left(\frac{1+b}{a}\right) - 4(\alpha - 1)\right\} \qquad (31)$$

which ϕ is a half of the angle between two side walls. Fig. 3 indicates the ne criterion for triangular channels, and it is obvious that the roll-wave mation in triangular channels cannot be compared with that in rectangular nnels.

The behavior of wave velocity of roll waves when the wave pattern becomes mately stable will be described next. By the use of the expressions for

progressive discharge rate and for the mean velocity of flow at the contr
section, the wave velocity is determined. Introducing the dimensionless wa
velocity by $V_W' = V_W/g\, q_o \cos\theta)^{1/3}$ for rectangular channels, in which q_o
the discharge per unit width for uniform flow, V_W' is

$$V_W' = \left\{\alpha - 1 + \sqrt{\alpha(\alpha - 1) + (1/N_F^2)}\right\}^{1/3} K'^{1/3} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (3$$

As the dimensionless progressive discharge rate K', in which

$$K' = K \frac{g \cos\theta}{V_W^3}$$

$$= \frac{1}{N_F^2} \left\{ \frac{\alpha - \sqrt{\alpha(\alpha - 1) + (1/N_F^2)}}{\alpha - (1/N_F^2)} \right\}^2 \left\{ 1 - \frac{\alpha - \sqrt{\alpha(\alpha - 1) + (1/N_F^2)}}{\alpha - (1/N_F^2)} \right\} . \; . \; . \; (3$$

is a function of α and N_F. so the dimensionless wave velocity is also e
pressed in terms of the momentum correction factor and Froude numbe

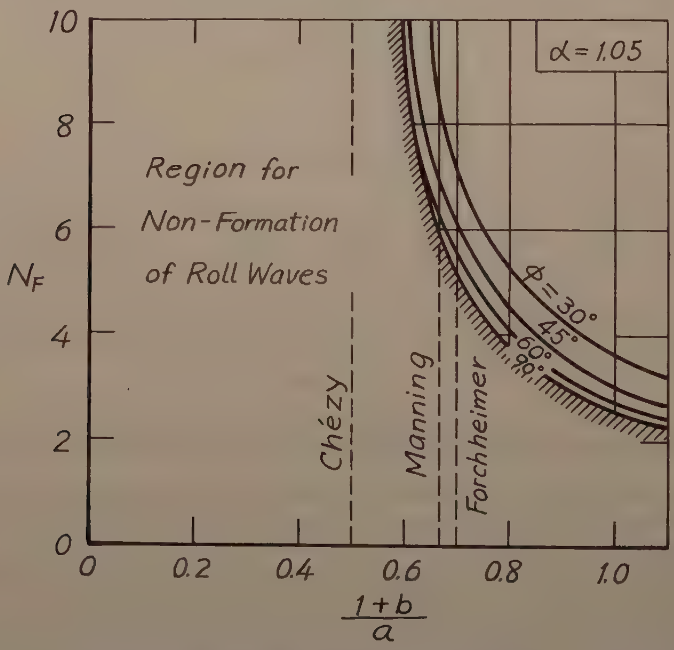


FIG. 3.—CRITERION FOR FORMATION OF ROLL WAVES IN TRIANGULAR CHANNELS.

Fig. 4 shows the behaviors of V_W' for both laminar and turbulent flows. In t
same figure, our experimental data are plotted with data of the author a
Binnie, and it is found that the experimental verification, to the theoretic
approach, can be made.

By the similar procedures, the behaviors of wave length, wave period, a
wave height as the hydraulic characteristics are determined. Introducing t

llowing dimensionless parameters of wave length, wave period and wave ight,

$$L' = L \tan\theta\,(g\cos\theta/V_w^2), \quad \text{(34a)}$$

$$T' = L'/V_w', \quad \text{(34b)}$$

d

$$D_b' = D_b\,(g\cos\theta/V_w^2), \quad \text{(34c)}$$

gs. 5-7 are graphically obtained by the numerical calculation, using the regoing relationships. The actual wave height at the frontal side is always

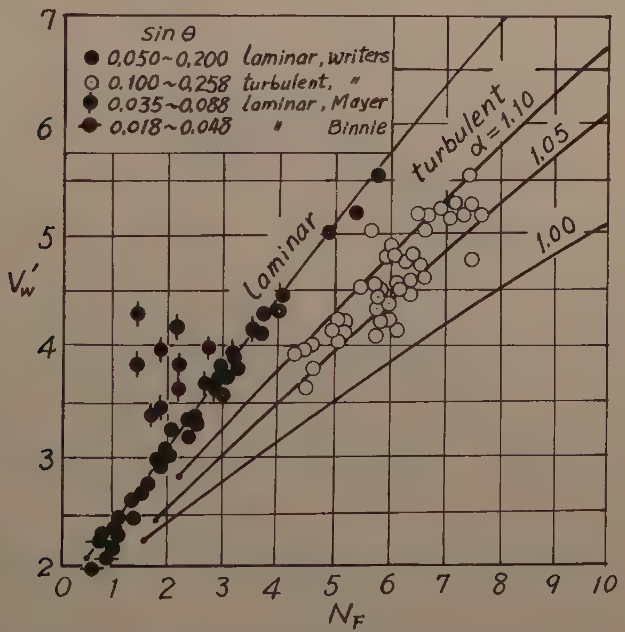


FIG. 4.—BEHAVIORS OF DIMENSIONLESS WAVE VELOCITY.

s than the calculated one, because the calculated wave profiles have sharp ve crests due to the discontinuous shock front, whereas the actual wave est is round probably because of the capillary effect and the existence of ticicities at the frontal side.

Other hydraulic characteristics in flow, due to the formation of roll waves, calculated in the same way. However, these are not concerned herein, and details of hydraulic characteristics in flow will be seen in the writers' er.[4-7]

Based on the results derived and the knowledge obtained by the writers ough the research works, the following 16 items are presented:

1. The author states that the roll wave is the result of interaction between surface tension and the gravity force in a slightly disturbed flow. How-

ever, the hydraulic characteristics of roll waves in laminar flow can be theo-retically analyzed as presented in the foregoing, without introducing the effec of surface tensions, and the results derived theoretically are satisfactorily i agreement with the experimental ones obtained by the writers and Binnie From this fact it may be concluded that the effect of capillary forces on th formation and the characteristics of roll waves is quite little or rather negli-gible practically, except in cases of extremely steep slopes and of very lo Reynolds number.

2. The author concludes that slug flows result from instability whic causes the transition from supercritical laminar to turbulent flow. Slug flows defined by the author, seem to be corresponding to incomplete roll waves a low Reynolds number in turbulent flow just changed from laminar to com-pletely turbulent through transition in its flow regime. The fact that roll wave and slug flows are not observed at the transition from laminar to turbulen

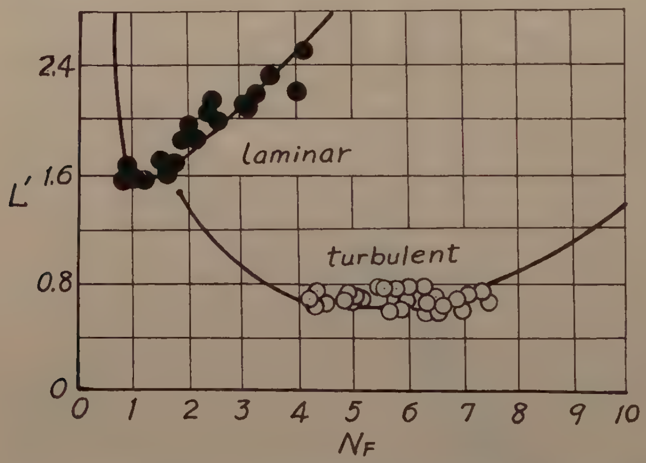


FIG. 5.—RELATION BETWEEN DIMENSIONLESS WAVE LENGTH AND FROUDE NUMBER.

flow is caused by the different nature of flow and thus the different laws resistance as described in the foregoing. Slug flows, therefore, should be ex-plained as kinds of roll waves which result from instabilities of flow itself, low Reynolds number in turbulent flow as well as roll waves in laminar flow

3. The author's Eq. 11 is questionable, whether it represents the criteri for formation of roll waves and slug flows or not. According to Eq. 11, ro waves are formed in laminar flow, when $N_F < 2$. However, the author's Fi 12 indicates that roll waves were observed even when $N_F > 2$, and the write also obtained the same results as shown in the figures illustrated in the fore-going. Consequently, $N_F = 2$ is not a criterion for formation of roll waves laminar flow.

4. The author applied the criterion for the instability of turbulent flow a the formation of turbulent roll waves,

$$S = 4\ g/C^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3$$

hich was derived by the use of the Chézy formula, to laminar flow. Since, owever, Eq. 35 was derived by putting C = constant, the author's Eq. 21 hould not be used to obtain Eq. 17a from Eq. 35.

5. In regard to the effect of surface tensions, the author suggests that the elocity of roll waves is expressed by

$$V_w = V + C_{min.} \quad \ldots\ldots\ldots\ldots (36)$$

nis relation is based on the idea that the wave length decreases and conersely the wave height increases with time, and ultimately the condition of inimum celerity C_{min} is attained. However, it is evident from Figs. 20 and that the wave velocity, the wave length, and the wave height actually inrease until the terminal values are reached with the lapse of time. This fact inconsistent with the author's suggestion.

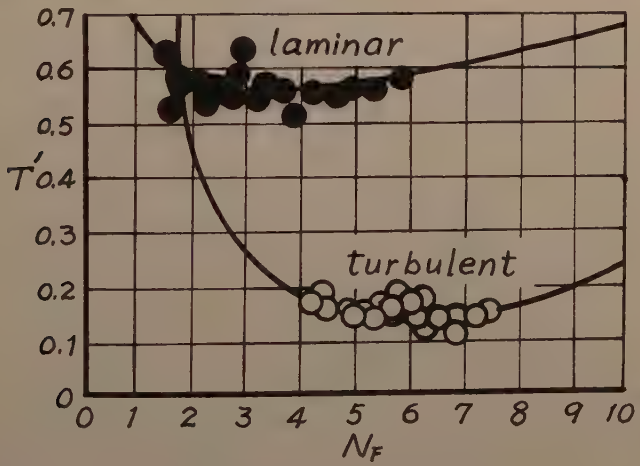


FIG. 6.—RELATION BETWEEN DIMENSIONLESS WAVE PERIOD AND FROUDE NUMBER.

6. For laminar flow, the author's Eq. 19 verifies that the plot of $N_F/S^{1/2}$ ainst R_D shown in Fig. 14 is correct as illustrated again in Fig. 18. In the me manner, since Eq. 19 is written as

$$N_F/\sqrt{S} = \text{const. } (g,\ \nu,\ \sigma/\rho) \cdot (N_w/S^{1/3})^{3/10} \quad \ldots\ldots\ (37)$$

e author's Eq. 15 is suggested to plot $N_F/S^{1/2}$ against $N_w/S^{1/3}$.

7. Graphical expressions for N_F against R_D in Fig. 12 and for N_F against , in Fig. 13 are the indication of the laws of resistance in each regime of w, and the hydraulic characteristics of roll waves and slug flows are not own. Therefore, the author's conclusion is not derived from these figures, t the surface tensions and viscous effects predominate in the phenomenon roll wave formation, and at higher Reynolds numbers, corresponding to the nditions which led to slug flows, viscous and surface tension effects deeased and, finally, became negligible.

According to the results of analysis by the writers, the viscous effect on characteristics of roll waves in laminar flow is introduced indirectly as

Froude number from the relation of Eq. 19 because all the dimensionles quantities of roll wave characteristics are functions of the Froude number, a shown previously. On the other hand, characteristics of roll waves (slug flow in turbulent flow are not influenced by the viscosity as far as Manning formula is used and its roughness coefficient is assumed independent on tl viscosity. Actually, however, the viscous effect may exist more or less if tl channel bed condition is a hydraulically smooth boundary.

8. If N_D and F_{cr} are put in a form of

$$N_D = \gamma F_{cr}^{\delta} \quad \text{. . . . . (3}$$

it is found after simple computation using the author's Eqs. 19 and 37 that tl value of δ should be 1/4 for laminar flow instead of 0.218 in the author's E 32.

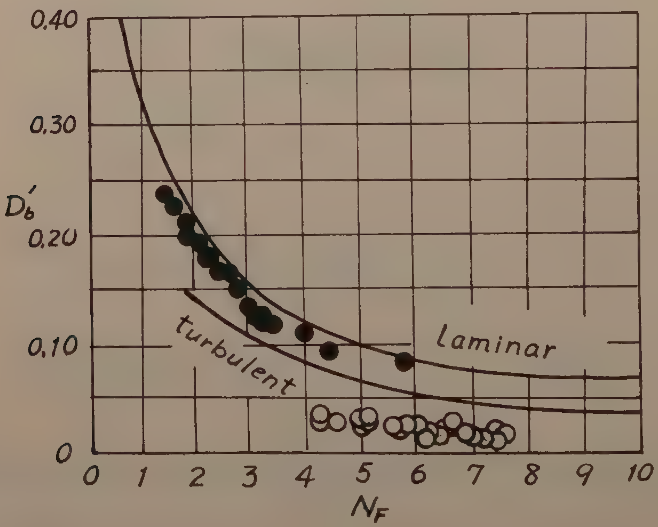


FIG. 7.—RELATION BETWEEN DIMENSIONLESS WAVE HEIGHT AND FROUDE NUMBER.

9. In obtaining the minimum slope 0.029 ~ 3% for the formation of r waves, the criterion to laminar flow

$$S \geqq \frac{12}{R_D} \quad \text{. . . . . (3}$$

was applied. This criterion is corresponding to

$$N_F \geqq 2 \quad \text{. . . . . (4}$$

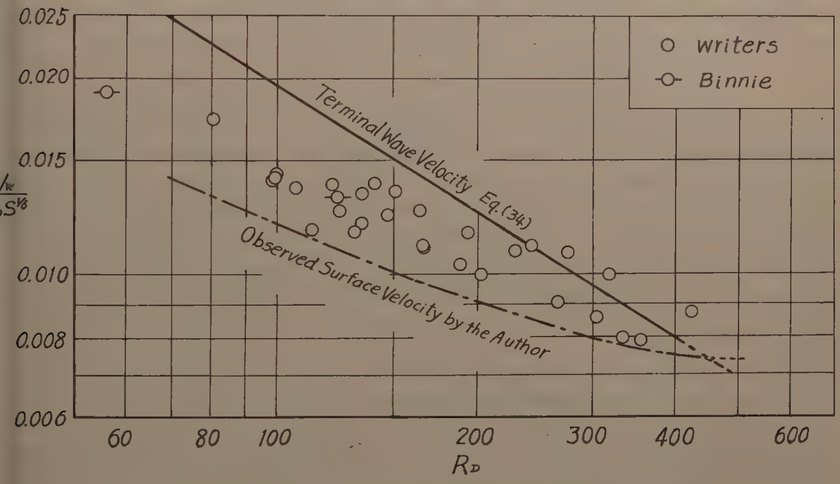
and based on

$$V + \sqrt{g\,D} \leqq V_s \quad \text{. (4}$$

wever, Fig. 17 indicates that roll waves develop even when $N_F < 2$ and it is ident that the latter is inconsistent with the author's idea for the roll wave mation.

10. In deriving the minimum slope 0.01 for the formation of slug flows, . 39 was also applied. At the Reynolds number 1,200, which the author used a lower limit of Reynolds numbers, the flow regime will be the transition m laminar to turbulent flow. Therefore, it is doubtful that Eq. 39, which s derived by using the law of resistance to laminar flow, can be applied to formation of slug flows.

11. The writers are much interested in Fig. 19 which shows the location of tiation of roll waves and slug flows. This figure indicates the fact that roll ves and slug flows cannot be formed in the transition from laminar to tur-ent flow, otherwise very long distances are needed for their development. is fact was already pointed out by the writers in their Fig. 1.



. 8.—PLOTS OF THE WRITERS' AND BINNIE'S DATA OF ROLL WAVE VELOCITY IN FORM PROPOSED BY THE AUTHOR.

12. The author suggests that the wave velocity of roll waves V_w is equal to C_{min}, in which C_{min} = 0.763 ft per sec. Although the value of V_w - V ob-ned from the experimental data is of the same order in magnitude as C_{min}, suggestion to the wave velocity is too intuitive and the effect of capillary given too much evaluation.

13. The writer's and Binnie's data of the wave velocity in laminar flow are sented in Fig. 8 in the same way as plotted in Fig. 24. It is found from the ure that the author's data are larger than the writer's and Binnie's, except Reynolds numbers of 280 to 420. This disagreement in addition to that in . 4 cannot be understood by the writers.

14. The ratio of the wave velocity V_w to the surface velocity V_s in laminar v is written, from Eq. 8, as

$$\frac{V_w}{V_s} = \left(\frac{V}{V_s}\right.\ \left(\frac{V_w}{V}\right)$$

$$= \frac{2}{3}\ \frac{6}{5} + \sqrt{\frac{6}{25} + \frac{1}{N_F^2}}$$

$$= \frac{4}{5} + \frac{2}{3}\sqrt{\frac{6}{25} + \frac{3}{S\ R_D}} \quad \ldots\ldots\ldots\ldots\ldots (4$$

This relation shows that the ratio V_w/V_s is a function not only of R_D but al of the slope S. Therefore, the slope should be introduced as a parameter Fig. 25. It is not difficult to find from Eq. 42 that the value of V_w/V is alway greater than unity and increases with decrease in R_D if the slope is constan This trend agrees with that shown in Fig. 25.

15. In turbulent flow, the ratio of the wave velocity (slug flows) to the su face velocity is expressed, from Eq. 6 if assuming $\alpha = 1$, as

$$\frac{V_w}{V_s} = \left(\frac{V}{V_s}\right)\left(1 + \frac{1}{N_F}\right) = \left(\frac{V}{V_s}\right)\left(1 + \frac{3}{S\ R_D}\right) \quad \ldots\ldots\ldots (4$$

Although it seems possible from Eq. 43 that the value of V_w/V_s becomes le than unity, which depends on the value of V/V_s, the results of his observatio which indicates that the surface velocity exceeds the wave velocity of sl flows, can not be understood. If so, when the flow with waves is transferred steady-state by moving the coordinate system at a constant wave velocity, t water surface in the virtual flow of water running upstream has to have velocity in the downstream direction. This phenomenon is possible only in t vicinity of the wave crest which results from the horizontal vortex due hydraulic jump.

By only this possible behavior of flow, the experimental results of sl flows cannot be interpretated analytically.

16. It may be rather better than the plot in Fig. 26, that the ratio of t wave height to the wave length D_h/L is plotted against Froude number with parameter of the slope because both $D_h\ g \cos\theta/V_w^2$ and $L \tan\theta\ g \cos\theta/V$ are functions of N_F as presented previously.

SPILLWAY DESIGN FOR PACIFIC NORTHWEST PROJECTS[a]

Discussion by B. Michel and A.R. Gagnon

B. MICHEL[1] and A.R. GAGNON,[2] A.M. ASCE.—This paper has brought up e of the oldest, but still one of the most discussed, problems of hydraulic gineering: the design of spillway crests.

The author should be commended for having presented his interesting views this problem and for the attempt to standardize the design of spillways.

In this last regard the writers agree fully with Mr. Webster that semicir-ar pier noses induce small flow contractions and that a 1 on 12 taper on the vnstream end of gate slots is generally one of the best means to lower cavi-ion risks at the slots.

However, the writers believe that a second thought should be given before ieralizing the process of designing spillway cress for 75% of the nominal id.

An outstanding contribution in that field has been made recently by R. moine[3] and the writers believe that he has given a very general basis for llway crest design. Mr. Lemoine proposes to adopt a cavitation factor for llway crests in the same manner as for the design of turbines:

This factor is

$$\sigma = \frac{\rho - \rho_v}{w \frac{v^2}{2g}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

vhich ρ_v is the water vapor pressure at the site, ρ represents the absolute iimum pressure on the spillway crest, $v^2/2g$ is the velocity head at the nt of minimum pressure, and w denotes the specific weight of water.

This factor takes into account, not only the actual permanent negative ssure in the dangerous area, but also the danger of having the velocity head nsformed into an additional negative pressure in low pressure nuclei ed-s, formed behind asperities of the spillway surface. Thus, the factor should er all conditions of cavitation on spillway profiles.

The design of any spillway can then be made with the following considera-s:

$\sigma = 0$: certain cavitation;

a August, 1959, by Marvin J. Webster.

1 Prof. of Hydr. Engrg., Laval Univ.

2 Lecturer in Fluid Mech., Laval Univ., Quebec City.

3 "Cavitation on Weir Crests," by R. Lemoine. Proceedings from 6th Meeting of the R, The Hague, 1955.

$\sigma = 1$: cavitation probably impossible, even if all the velocity head w transformed into additional negative pressure; and

$\sigma = 0.5$: acceptable risk of cavitation if this working condition occu rarely and if the spillway surface is very smooth.

For a standard ogee spillway it is easy to know the minimum pressure the crest,[4,5,6] the velocity head, and to design the spillway with the help the cavitation factor. It is indeed interesting to note that the foregoing refe ences show that, for an overdesign of 75%, the cavitation factor, after a "cr tical design head," is invariably zero while the minimum pressure becom equal to the vapor pressure. It might be also noticed that the classical for is not generally the one that will give the highest discharge coefficient for certain value of the cavitation factor. Model tests must be made in each cas

The model results given in Mr. Webster's paper, for the Chief Jose Spillway, show a minimum pressure of -23 ft of water with no piers and -9 with piers. These results are somewhat higher than those obtained on simil spillway profiles by many investigators. This could probably be explained the fact that very small differences in the geometry of each project have important effect on pressure values, and also by the very high difficulty i volved in measuring the peak of rapidly varied pressure distribution on scale model. The writers do not believe that the results for this spillway a representative of all those of the same type.

Taking the last value for the minimum pressure of -9 ft with an operati head of 54 ft it is found that the cavitation factor would then be 0.38, which somewhat lower than could be permissible with Mr. Lemoine's theory.

The author was fully aware of some cavitation risks and states that due the low predicted frequency of the maximum discharge, the cost saved in i itial construction more than offsets the cost of infrequent repairs resulti from damage due to negative pressure.

The writers agree with him and this could easily be taken into account choosing, in each case, an accepted value of the cavitation factor. The write strongly believe that this would be more logical than to accept a percenta head design irrespective of the value of the head itself, the actual shape a pressures on the crest.

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4 "Untersuchugen an Überfällen," by O. Dillmann. Mitteilungen des hydraulisc Institus, Munich #7, 1933.

5 "Engineering Hydraulics," Hunter Rouse (Editor), Chapter VIII.

6 "Model Research on Spillway Crest," by Rouse and Reid. Civ. Engrg., Vol No. 1, 1935.

FLOOD CONTROL ASPECTS OF CAUCA VALLEY DEVELOPMENT[a]

Discussion by Julio Escobar-Fernandez

JULIO ESCOBAR-FERNANDEZ.[1]—Three aspects from the paper by ssrs. Kirpich and Ospina could be considered. First, the significance and ortance, in the development of the so-called underdeveloped countries, of establishment of regional autonomous corporations of development; sec-, the technical difficulties that have to be faced by engineers interested in er problems in such countries, where hydrological data is invariably rce, or of doubtful value and inconsistent; and third, the technical features big hydroelectric project, fleetingly mentioned in the paper, which could developed by diverting the waters of the Cauca River to the Pacific coast, stem that could be classified as one of the important energetic possibili-, not only of Latin America but of the world, with the particular advantage ts proximity to the sea, a feature that makes it especially attractive for establishment of heavy electrochemical industries.

lthough the three aspects might be of general interest, due to limitations pace, only the first one will be discussed, at present. Some of the general nciples governing the policies and the organization of regional corporations oted to local development are set forth, and some emphasis is placed on importance of such bodies as helpful instruments in the execution of wider elopment programs, usually national in scope. The timeliness of the sub- seems especially relevant today (1960), when better management for nomic improvement constitutes a daily concern for the governments in the developed countries.

Colombia is a country with rich natural resources that can be developed to ate an economically solid nation, thereby raising the standard of living of people. To reach these two objectives, in time and without waste of effort, s considered that a plan for the development and effective use of those re-rces is indispensable, as is the implementation of the plan. In Colombia, Central Government has already launched such a program and the recent-rganized Planning Unit, directly responsible to the President of the Re-ic, is a fundamental step towards the coordinated execution of the plans have been already formulated or which are under study. As a starting t, a better orientation of government investments is considered one of the nary policies of development, because such investments as in transporta-, energy production, irrigation and public utilities are or should be the dation on which a solid economy must rest.

n the other hand, the expansion of public enterprise, which constitutes the r fundamental basis on which the progress of nations depends, must be

September, 1959, by Phillip Z. Kirpich and Carlos S. Ospina.
Escobar, Venegas y Rodriguez, Ingenieros Ltda., Bogota, Colombia.

stimulated not only by the facilities provided by those public investments, b by the creation, through government measures, of a minimum of general co ditions favoring private investment.

As one of its immediate tasks, the new Planning Unit will therefore have establish a balance between all resources available—economical, physical a human—and the many needs of the country. From this overall picture, a mo or less rigid schedule of priorities must be determined, in order to establi how and where such resources can be spent most usefully and effectively.

Once a general plan for the whole country has been defined, the next st should be to project it on a sectorial basis, to use the economist's jargon, on a regional basis, if the characteristics of the country so allow, as is t case in Colombia.

A program of integrated regional development, such as the one undertak in Colombia in the valley of the Cauca River has, among others, the followi advantages:

1. It permits the establishment of a kind of experimental field where se eral techniques of development can be tested. In this way, the selected zo becomes a demonstration area, where direct experience is the best school;

2. financial problems may be of a reduced magnitude in comparison with nation-wide program of development.

3. administrative problems are also reduced; and,

4. in developing a particular area, the economic benefits spread thoroug out the country.

When in 1954, David Lilienthal, mentioned by the authors of the paper und discussion, recommended to the Government of Colombia the creation of autonomous regional authority,[2] as a first stage in its wider program of d velopment, he based his recommendations on several general principles th should guide this type of body, principles that can be resumed as follows:

The initiative for a general program must originate in the region itself. is therefore indispensable that people in the area be willing to cooperate the plans, and that enthusiasm for innovation and improvement be general.

Second, the approval and cooperation of the Central Government is al necessary. The Government should be disposed to frame the program w some autonomy, through the creation of an agency or public corporation d voted to that purpose. This agency should be provided with sufficient admi istrative authority, and with well defined functions and aims. To better defi the status of that corporation, it may be useful to quote the words of Preside Roosevelt when he asked Congress to create the TVA in 1933 (quoted by M Lilienthal,: The authority should be, he said "a corporation clothed with t power of government but possessed of the flexibility and initiative of a priva enterprise."

Third, the financial assistance of the Government or of any internation organization, such as the World Bank.

Fourth, the creation of a local economic development corporation to pr mote and stimulate private enterprise in its own development programs.

Finally, it is advisable that the selected region for an experiment of th sort, especially if it is the first to be undertaken in the country, should be c

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[2] "Recommendations on the Establishment of Regional Development Authorities the Republic of Colombia," by David E. Lilientahl, June 25, 1954.

ere the benefits obtained through the techniques of planning, are more or
s immediate, in order to overcome the natural tendency of people to be
ptical or even hostile to programs that require a long time to produce
inite results.

All of these conditions were satisfied by the Colombian region of the Cauca
ley, and the CVC, the autonomous corporation created in 1955, is beginning
show results in a way that may be considered a real success.

FRICTION FACTORS IN CORRUGATED METAL PIPE[a]

Discussion by A.R. Chamberlain and Nicholas Bilonok

A.R. CHAMBERLAIN,[1] A.M. ASCE.—The authors are to be commended for ir very careful work in obtaining design information on friction factors for ge-diameter corrugated pipes. These data and results presented by the hors will undoubtedly be used by design engineers for several decades, in cases where pipes of the nature tested are used. This paper represents the ult of a tremendous amount of effort on the part of Army Corps personnel.

The writer wishes to extend the data presented by the authors on the basis clear water, full pipe flow data obtained during 1953-56 in the Hydraulics oratory, Colorado State University, Fort Collins, Colo. These data were ained incidental to certain basic research studies on nominally 12-in. dieter smooth, standard corrugated and helical corrugated pipes.

Each of the types of corrugated metal pipe sections tested were 100-ft long, on a zero slope. Both corrugated pipes had a clear inside diameter of 1 ± 0.03 in., as determined from ten trials on each pipe.

The water was recirculated in a closed system. The discharge was measd by means of a sharp-edged orifice which had been calibrated volumetriy while in place. Discharges were determined, for each discharge-controle setting, from the orifice calibration curve using the arithmetic average our separate readings of the differential head across the orifice.

Hydraulic gradient was determined by means of water piezometers located 0-ft intervals along the pipeline. The piezometer openings into the pipeline e 3/64-in. in diameter. For each discharge, each piezometer was read times and the arithmetic average used in hydraulic gradient computas. Upstream end (for 20 pipe diameters) and downstream end (for 10 pipe neters) piezometers were generally ignored, though recorded, in hydraugradient studies. The piezometers were located on the crest of the corations (viewer inside the pipe) after preliminary investigations of the ef- of piezometer location along the corrugation, comparable to those of the or's.

The procedure was to first read the orifice manometer, then the piezomebank and next the orifice manometer again, until each had been read four es during a period of about 1 hr. Next the discharge valve setting was ged, 15 to 20 min. allowed for the flow to stabilize and the next set of to be obtained.

Fig. C1 shows the two boundaries tested and reported in this discussion. standard corrugated pipe was zinc coated close-riveted construction.

September, 1959, by M. J. Webster and L. R. Metcalf.

Acting Dean, Coll. of Engrg. and Chf., Engrg. Research, Colorado State Univ., Fort ins, Colo.

The corrugations had a pitch of 0.222 ft along the axis of the pipe. The amp
tude of the corrugations from the mean was 0.0208 ft. The crests and troug
were circular arcs with an included angle of approximately 53.73 deg anc
radius of 0.0573 ft. Straight tangent sections joined the circular arcs.

The helical corrugated pipe was zinc coated. It had a continuous lock se
joint. The corrugations had a pitch or wave length of 0.167 ft normal to
axis of the corrugation. The amplitude of the helical corrugations was 0.01
ft, measured from the mean elevation. To complete the corrugations, circu
arcs with a radius of 0.0370 ft were joined by straight tangent sections. T

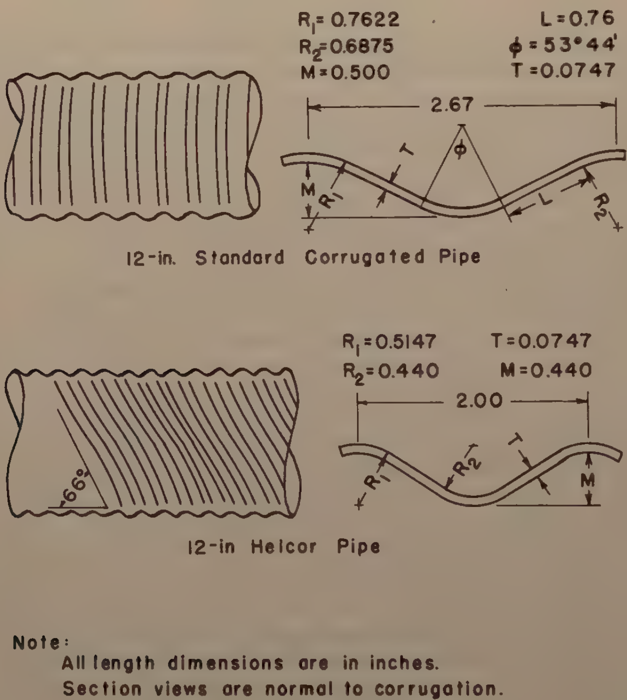

FIG. C1.—DETAILS OF BOUNDARY FORMS.

helix angle, measured between a line drawn along the outer extremity of
pipe parallel to the pipe axis and a line parallel to a corrugation, was 66 de

Garde's data were obtained with clear water full pipe flow after the pi
lines had operated several hundred hours, transporting 0.2 mm mean diame
sand at varying concentrations. Chamberlain's data, designated by a cross,
the Helcor pipe only, were taken just prior to Garde's. The remaining da
all taken by Chamberlain, were recorded with clear water full pipe flows
new pipe.

Derived quantities Re, f, and V were either computed directly or read fr
graphs prepared for the purpose. In either case, the pipe diameter emplo
in the analysis was 1.00 ft, slightly less than the inside clear diameter. T
magnitude of the diameter was used because (a) it was very convenient,

re important (b) it was felt that most engineers applying the data would omatically use 1.00 ft in all their design computations.

Tables 1 and 2 summarize the full pipe flow data obtained, admittedly not y extensive, but hopefully of some use to the profession. These data are esented in Fig. C2, for both the standard corrugated and helical corrugated, a manner comparable to the hor's Fig. 10. They are also en in Fig. C3 in the same way the author's Fig. 11.

Fig. C3 indicates that a recmendation for new 1-ft diamr helical corrugated pipe, of 0.040, is appropriate. This ne figure shows that f is not a ction of Reynold's number er the range tested. This may partially explained by the fact t the corrugations are not rmal to the direction of flow in pipe.

The writer is unable to say Garde's data, previously unlished, have such a wide scat. This is particularly true of helical corrugated pipe data, ce the pipeline was not in use ing the interval between amberlain's and Garde's usede tests.

Perhaps the most important nt to note is that the dashed tion at the smaller diameter of the author's Fig. 17 can be ended by plotting in for standcorrugated pipe the tentativeecommended point f = 0.12, at 1.0 ft. Also to be plotted on . 17 for corrugated pipe is the nputed (from f = 0.120) 0.0255 at a pipe diameter of 1 ft. The data are insufficient to determine if a function of Reynold's number.

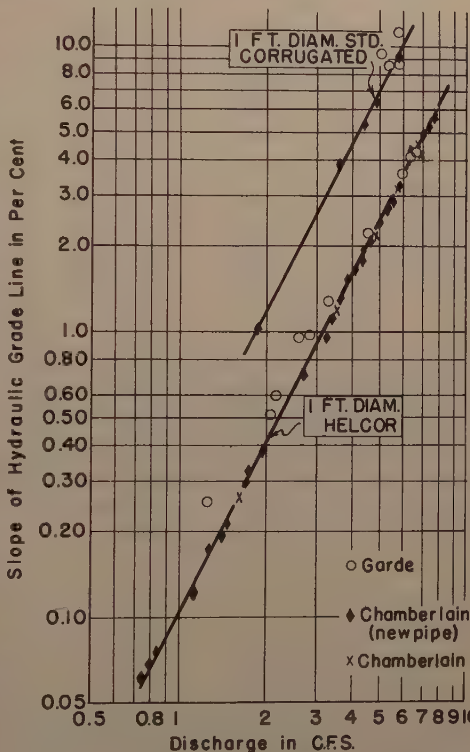

FIG. C2.—DISCHARGE VERSUS HYDRAULIC GRADIENT FOR 1-FT. DIAMETER HELCOR AND CORRUGATED PIPES.

Fig. C4 graphically describes the variation of the total differential head veen crest and trough piezometers for the standard corrugated pipe tests function of Reynold's number. Perhaps more important, however, is the , also in Fig. C4, of the ratio of the hydraulic gradients as determined by ometers at the crest(s) and at the trough(s) of the corrugations, as viewed n inside the pipeline. This plot substantiates that the author's and the er's method of determining hydraulic gradient from a set of piezometers, ted consistently at the same relative location on the corrugations along pipeline, is sufficiently accurate for engineering purposes over the range eynold's number incorporated in the tests.

TABLE 1.—SUMMARY OF TEST RESULTS FULL PIPE FLOW 1-FT-DIAMETER CORRUGATED PIPE[a]

V, in feet per second (1)	Q, in cubic feet per second (2)	S, in % (3)	Water Temperature in C° (4)	Re x 10^{-3} (5)	(
6.37[a]	5.00	9.24	19.5	590	0.
6.65[a]	5.21	8.50	17.7	583	0.
7.26[a]	5.70	10.70	19.0	655	0.
7.26[a]	5.70	8.50	19.2	660	0.
				Average	0.
2.38[b]	1.87	1.00	27.1	263	0.
4.45[b]	3.54	3.80	22.1	430	0.
5.30[b]	4.32	5.20	21.6	510	0.
5.98[b]	4.78	6.24	23.3	605	0.
7.23[b]	5.80	9.00	25.3	770	0.
				Average	0.

[a] Data from R.J. Garde, "Sediment Transport Through Pipes," Thesis in partial f fillment of the requirements for the Master of Science Degree, Colorado State Univ sity, Fort Collins, Colorado, December 1956.

[b] Data from A.R. Chamberlain, "Effect of Boundary Form on Fine Sand Transpor Twelve-Inch Pipes," Dissertation in partial fulfillment of the requirements for Doctor of Philosophy Degree, Colorado State University, Fort Collins, Colorado, J 1955.

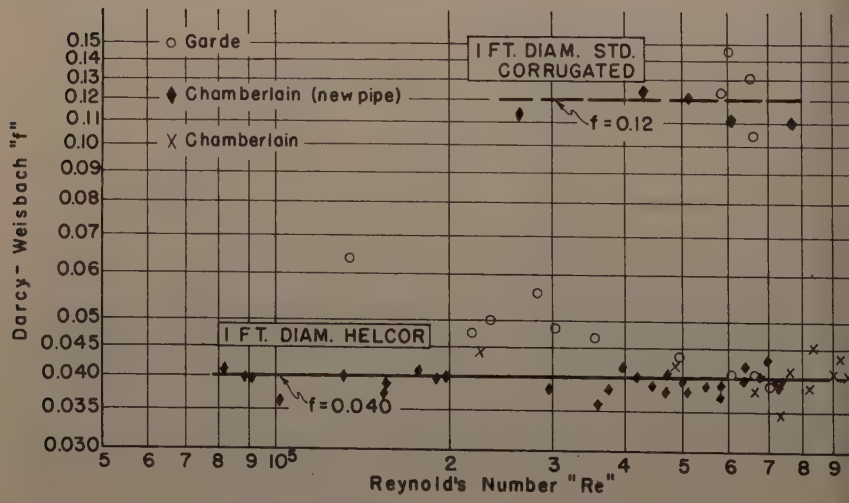

FIG. C3.—DARCY-WEISBACH COEFFICIENT VERSUS REYNOLD'S NUMBER FOR 1 FT. DIAMETER HELCOR AND STANDARD CORRUGATED PIPES.

TABLE 2.—SUMMARY OF TEST RESULTS FULL PIPE FLOW
1-FT-DIAMETER HELICAL CORRUGATED PIPE

V, n feet r second (1)	Q, in cubic feet per second (2)	S, in % (3)	Water Temperature in °C (4)	Re x 10^{-3} (5)	f (6)
1.59[a]	1.25	0.25	16.5	135	0.0640
2.62[a]	2.06	0.50	15.8	218	0.0478
2.76[a]	2.17	0.59	16.7	235	0.0500
3.31[a]	2.60	0.95	17.0	284	0.0559
3.58[a]	2.81	0.96	16.2	302	0.0483
4.17[a]	3.27	1.27	16.7	356	0.0470
5.73[a]	4.50	2.20	17.0	493	0.0435
7.50[a]	5.89	3.56	14.6	608	0.0408
7.97[a]	6.25	4.02	15.9	666	0.0408
8.35[a]	6.56	4.22	16.3	705	0.0390
0.98[b]	0.75	0.062	15.2	82	0.0413
1.05[b]	0.80	0.068	15.4	89	0.0396
1.10[b]	0.84	0.074	15.0	91	0.0394
1.46[b]	1.13	0.12	14.8	102	0.0368
1.63[b]	1.27	0.17	14.6	132	0.0401
1.77[b]	1.40	0.19	17.4	155	0.0390
1.88[b]	1.47	0.21	14.4	153	0.0375
2.08[b]	1.63	0.26	26.4	227	0.0446
2.16[b]	1.70	0.29	17.4	190	0.0397
2.22	1.73	0.32	14.2	177	0.0412
2.45[b]	1.92	0.37	14.0	197	0.0400
3.43[b]	2.70	0.70	17.4	298	0.0384
4.10[b]	3.25	0.94	17.3	358	0.0360
4.29[b]	3.40	1.11	17.3	375	0.0385
4.45[c]	3.53	1.18	27.0	485	0.0422
4.52[b]	3.60	1.28	17.2	420	0.0404
4.80[b]	3.80	1.50	17.1	395	0.0419
5.17[b]	4.10	1.62	17.0	445	0.0391
5.41[b]	4.30	1.74	17.0	470	0.0382
5.45[b]	4.32	1.90	17.0	472	0.0412
5.80[b]	4.60	2.00	17.5	510	0.0383
6.00[c]	4.78	2.15	27.1	660	0.0382
6.23[b]	4.95	2.39	14.0	500	0.0397
6.61[b]	5.23	2.65	17.5	580	0.0391
6.80[b]	5.43	2.80	13.8	548	0.0390
7.23[c]	5.70	3.15	26.2	765	0.0418
7.24[b]	5.82	3.23	17.5	635	0.0397
7.39[c]	5.80	3.24	22.5	735	0.0350
7.40[b]	5.90	3.20	13.6	580	0.0376
7.95[b]	6.30	4.25	17.5	700	0.0433
8.10[b]	6.42	4.29	13.4	640	0.0422
8.21[c]	6.60	4.09	27.0	900	0.0411
8.40[c]	6.70	4.43	28.5	950	0.0406
8.40[c]	6.70	4.20	21.3	820	0.0392
8.46[b]	6.80	4.20	17.4	740	0.0396
8.68[b]	6.95	4.79	13.3	675	0.0409
9.20[b]	7.35	5.20	13.1	720	0.0396
9.51[b]	7.62	5.50	13.0	735	0.0392
9.72[c]	7.63	5.60	17.0	840	0.0456
9.80[c]	7.70	5.80	20.7	925	0.0438

[a] Data from R.J. Garde, "Sediment Transport Through Pipes," Thesis in partial fulfillment of the uirements for the Master of Science Degree, Colorado State University, Fort Collins, Colorado, ember 1956.
[b] Data from A.R. Chamberlain, "Effect of Boundary Form on Fine Sand Transport in Twelve-Inch es," Dissertation in partial fulfillment of the requirements for the Doctor of Philosophy Degree, orado State University, Fort Collins, Colorado, June 1955.
[c] Data obtained by Chamberlain after pipeline had been used to transport 0.2 mm sand for several dred hours.
[d] Arithmetic average, all Chamberlain data, f = 0.0400.

Acknowledgements.—The basic research studies were supported by Armc Drainage and Metal Products, Inc., Tau Beta Pi Association (through a fellov ship), and Colorado State University.

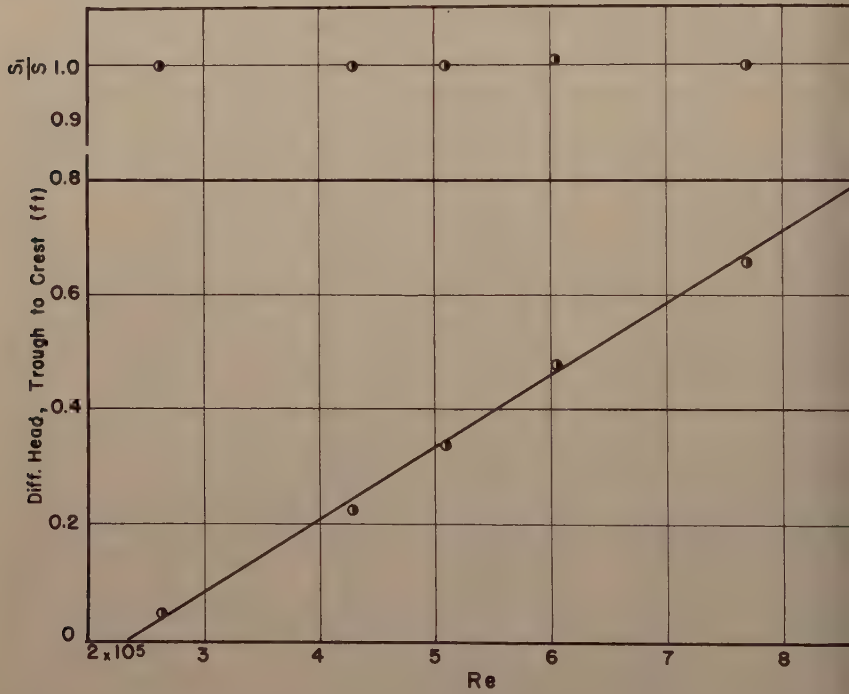

FIG. C4.—RATIO OF HYDRAULIC GRADIENT ON CORRUGATION HIGHS TO THAT C CORRUGATION LOWS, AND DIFFERENCE IN HEAD BETWEEN HIGHS AND LOW VERSUS REYNOLDS NUMBER FOR 12-IN. CORRUGATED PIPE.

NICHOLAS BILONOK.[2]—The most important contribution by the authors the presentation of data from certain carefully conducted experiments on fu size, large corrugated metal pipes, to determine friction factor "f" and abso lute roughness "n." The accuracy depends on a correct evaluation of "n," t well known coefficient of absolute roughness in the Manning formula, whi depends primarily on the surface condition of the perimeter of a pipe or ope channel.

Because of the complex nature of the friction factor "f" various attemp have been made to group all the roughness dimensions together into a sing coefficient of absolute roughness "n" for simplicity.

The writer is particularly interested in this paper as he had independent investigated the relationship between friction factor "f" and absolute roug ness "n," for many years.

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[2] Civ. Engr., Engrg. Div., Corps of Engrs., U.S. Army, Philadelphia, Pa.

The values of Q, S, H_f, L and D have been measured in experiments and the iction factor "f" has been found to be given by

$$f = \frac{H_f \; D \; 2 \; g}{L \; V^2} \qquad \text{(1)}$$

$$f = \frac{8 \; g \; R \; S}{V^2} \qquad \text{(2)}$$

he equation for the roughness coefficient "n" for the Manning formula is

$$n = \frac{1.486 \; R^{2/3} \; S^{1/2}}{V} \qquad \text{(3)}$$

q. 3 may be written from the friction factor "f" by the relationship

$$n = \frac{R^{1/6} \sqrt{f}}{10.8} \qquad \text{(4)}$$

q. 4 can be written as

$$f = \frac{116.57 \; n^2}{R^{1/3}} \qquad \text{(5)}$$

substituting $R = D/4$

$$f = \frac{185 \; n^2}{D^{1/3}} \qquad \text{(6)}$$

rom a review experiments data presented in the paper the writer has comled Table 1 to relate the friction factor "f" to Manning's roughness coeffients "n" ranging from 0.010 to 0.025.

The values of "f" computed from Eq. 6 are in agreement with those obned by experiments for pipes 3 ft, 5 ft, and 7 ft in diameter, and also with ble V in Reference 3.

Another quantity of importance that is the Reynold's number:

$$\mathbf{R} = \frac{V \; D}{\nu} \qquad \text{(7)}$$

The exact form and position of "f" versus Reynold's number for a given e diameter and roughness is not clear (Fig. 11), and close correlation reires more study.

The practical value of the Reynolds number is that it indicates the degree turbulence in flowing water. For a given size of circular pipe the velocity the major variable and the Reynolds number increases as the velocity of w increases.

The author's Tables A and B indicate that the Reynolds number varies velocity of flow only. The water temperature changes have no significance.
Investigations of friction losses for Part-Full flow, in Reference 3, ha found that "For uniform subcritical flow, the Manning coefficient did not inc

TABLE 1.—VALUES OF f x 10^3

D, inches \ n	0.010	0.011	0.012	0.013	0.015	0.018	0.021	0.024	0.0
6	23.3	28.2	33.6	39.4	52.4	75.5	102.7	134.2	145
8	21.2	25.6	30.5	35.7	47.6	68.5	93.2	121.8	132
10	19.7	23.8	28.4	33.3	44.3	63.8	86.9	113.5	123
12	18.5	22.4	26.6	31.3	41.6	59.9	81.6	106.6	115
15	17.2	20.8	24.7	29.0	38.6	55.6	75.8	98.9	107
18	16.2	19.6	23.4	27.4	36.5	52.5	71.6	93.4	101
24	14.6	17.8	21.1	24.8	32.8	47.5	64.8	84.5	91
30	13.6	16.5	19.6	23.0	30.0	44.0	60.0	78.3	85
36	12.8	15.5	18.5	22.0	28.7	41.6	55.6	74.0	80.
42	12.2	14.7	17.5	21.0	27.0	39.4	53.7	70.0	76.
48	11.6	14.1	16.7	20.0	26.0	37.6	51.3	67.0	73.
54	11.2	13.6	16.1	19.0	25.0	36.3	49.4	64.5	70.
60	10.8	13.1	15.6	18.0	24.0	35.0	47.7	62.3	68.
72	10.2	12.3	14.6	17.0	22.7	32.9	44.8	58.5	63.
84	9.7	11.7	13.9	16.4	21.6	31.3	42.7	55.7	60.
96	9.2	11.2	13.3	15.6	20.6	30.0	40.8	53.3	58.
108	8.9	10.8	12.8	15.0	19.8	28.8	39.2	51.2	55.
120	8.6	10.4	12.4	14.5	19.3	27.8	37.9	49.4	53.
132	8.3	10.1	12.0	14.1	18.7	27.0	36.7	47.9	51.
144	8.1	9.8	11.6	13.6	18.2	26.2	35.6	46.5	50.

cate a systematic variation with the Reynolds number or pipe size within th accuracy of experimental data."

It can be recognized that several factors might have affected the accurac of the test results and the computations.

REVISED COMPUTATION OF A VELOCITY HEAD WEIGHTED VALUE[a]

Discussion by Steponas Kolupaila, Israel H. Steinberg
and William C. Peterson

STEPONAS KOLUPAILA.[1]—Presentation of this paper proves that the oblem of the correction factor, known as a Coriolis' coefficient, is still ive, and still not entirely clear. The purpose of this factor is to correct velocity head in the Bernoulli equation for different velocities across the ction.

The basic expression given under 10, p. 72 is correct, except for an obus typographic omission: ΔQ is to be understood instead of Q in the nurator. This formula can give an accurate solution when a stream is divided a large number of small areas with measured velocities. This work is re reasonably performed by a graphical integration with a planimeter, minating those "astronomical" numbers. Application of this formula for eral parts of a cross section with assumed coefficients is a half measure. e value obtained by the authors (1.19) seems to be too low for an open chan- with overflooded banks.[2] The river gagings by two point or one point thods, although more than sufficient for the routine discharge measurent, are positively inadequate for the determination of the Coriolis coeffint. Two integrations are necessary in an open channel, along the verticals, l across the channel, and more points must be taken on every vertical.

ISRAEL H. STEINBERG,[3] F. ASCE.—The sample computations presented he article, and which are patterned after a procedure given in the Engiring Manual of the Corps of Engineers, still contain some of the objectione, relatively large, figures required to obtain a relatively small value of ocity head. For this reason the writer has adopted a number of alternate cedures in order to further reduce the necessity of dealing with large and ieldly values. One of the methods utilizes a weighted area curve for the tion. The equation is:

$$A_W = A_c \left\{ \frac{1}{\left[\left(\frac{C_c}{C}\right)^3 + \left(\frac{C_o}{C}\right)^3 \left(\frac{A_c}{A_o}\right)^2 \right]^{1/2}} \right\} \quad \ldots\ldots\ldots \quad (1)$$

a September, 1959, by J.M. Lara and K.B. Schroeder.
1 Prof. of Civ. Engrg., Univ. of Notre Dame, Notre Dame, Ind.
2 "Open-Channel Hydraulics," by V.T. Chow. McGraw-Hill, 1959, p. 28.
3 Asst. Chf., Planning and Reports Branch, U.S. Army Engr. Dist., San Francisco, ps of Engrs.

in which A_w is the weighted area, A_c and A_o are respectively, the channel a overbank areas, and C_c and C_o are, respectively, the channel and overba conveyance factors of the form:

$$\frac{1.486}{n} \frac{A \ R^{2/3}}{L^{1/2}} = \frac{K}{L^{1/2}} \quad \ldots\ldots\ldots\ldots\ldots$$

in which C is the total conveyance factor for the section $(= C_c + C_o)$, and L the reach length.

Eq. 1 was developed for flow line problems involving separation of the se tion into two parts; namely, channel and overbank. The equation can be read expanded to include any desired number of components. Taking the reciproc of Eq. 1 and squaring gives:

$$\left(\frac{1}{A_w}\right)^2 = \left(\frac{1}{A_c}\right)^2 \left[\left(\frac{C_c}{C}\right)^3 + \left(\frac{C_o}{C}\right)^3 \left(\frac{A_c}{A_o}\right)^2\right]$$

$$= \left(\frac{C_c}{C}\right)^3 \left(\frac{1}{A_c}\right)^2 + \left(\frac{C_o}{C}\right)^3 \left(\frac{1}{A_o}\right)^2 \quad \ldots\ldots\ldots\ldots$$

Eq. 3 applies, also, for any number of components and can be written in t more generalized form of

$$\left(\frac{1}{A_w}\right)^2 = \left[\left(\frac{C_1}{C}\right)^{3/2} \left(\frac{1}{A_1}\right)\right]^2 + \left[\left(\frac{C_2}{C}\right)^{3/2} \left(\frac{1}{A_2}\right)\right]^2$$

$$+ \left[\left(\frac{C_3}{C}\right)^{3/2} \left(\frac{1}{A_3}\right)\right]^2 + \ldots \quad \ldots\ldots\ldots\ldots$$

If L is constant throughout, the K-values can be substituted for correspondi C-values.

Eq. 4 can be further modified in more convenient forms for computi either (a) the weighted velocity head, h_v, or (b) the alpha coefficient, α, terms of total section area. To obtain h_v, both sides of Eq. 4 are multipli by $(Q/8.03)^2$, resulting in

$$h_v = \left(\frac{Q}{8.03 \ A_w}\right)^2 = \left[\left(\frac{C_1}{C}\right)^{3/2} \frac{Q}{8.03 \ A_1}\right]^2 + \left[\left(\frac{C_2}{C}\right)^{3/2} \frac{Q}{8.03 \ A_2}\right]^2$$

$$+ \left[\left(\frac{C_3}{C}\right)^{3/2} \frac{Q}{8.03 \ A_3}\right]^2 + \ldots \quad \ldots\ldots\ldots\ldots$$

Application of Eq. 5 in computing the weighted velocity head is illustrat in Table 1 using the author's values for section 2. The values in Cols. 7 and were obtained with simple settings of the slide rule.

TABLE 1

	L	Q	K_d	$\left(\frac{K_d}{K}\right)^{3/2}$	$\frac{Q}{8.03\ A_d}$	h_v
	(4)	(5)	(6)	(7)	(8)	(9)
3	1110	10,800	66,400	0.255	1.511	0.149
9		$\frac{Q}{8.03}$ =	67,000	0.258	2.295	0.352
0			5,240	0.006	9.000	0.003
4		134.8	26,300	0.062	2.280	0.020
6			164,940			0.524

f the value of α is desired, Eq. 4 is multiplied by A^2, the total sectional a, to give:

$$= \left(\frac{A}{A_w}\right)^2 = \left[\left(\frac{C_1}{C}\right)^{3/2} \frac{A}{A_1}\right]^2 + \left[\left(\frac{C_2}{C}\right)^{3/2} \frac{A}{A_2}\right]^2 + \left[\left(\frac{C_3}{C}\right)^{3/2} \frac{A}{A_3}\right]^2 + \ldots\ldots \quad (6)$$

TABLE 2

$\left(\frac{K_d}{K}\right)^{3/2}$	$\frac{A}{A_d}$	α
(7)	(8)	(9)
0.255	2.495	0.405
0.258	3.785	0.956
0.006	14.850	0.008
0.062	3.750	0.054
		1.423

omputations are the same as for the mple above except for Cols. 8 and 9. able 2 only Cols. 7, 8 and 9 are vn for brevity.

or the assumed value of Q = 10,800,

$$h_v = 1.424 \left(\frac{10800}{8.03 \times 2226}\right)^2 = 0.521$$

ILLIAM C. PETERSON,[4] M. ASCE.—Computation of an approximation of velocity head correction factor, α, by the fraction

$$\frac{K_1^3/A_1^2 + K_2^3/A_2^2 + \ldots K_n^3/A_n^2}{(\Sigma K)^3/(\Sigma A)^2}$$

ssentially a simple slide-rule operation, quickly accomplished when one mes familiar with the "feel" of the magnitude of the values involved. niques, which are largely personal, can abbreviate the consequent "as-omical values" and can also provide judgement in retaining the significant es of the computation.

Hydr. Engr., U.S. Geol. Survey, Los Angeles, Calif.

In the Appendix of the author's paper the expression for friction head lo
h_f, is written as

$$h_f = f + h_{v_1} - h_{v_2} - h_t$$

where

$$h_t = m\ (h_{v_1} - h_{v_2})$$

The authors neglect to state, however, that the quantity in parentheses mu
always be positive. For an expanding reach, $h_{v_1} > h_{v_2}$, the quantity in pare
theses is positive, and

$$h_t = m\ (h_{v_1} - h_{v_2})$$

Thus

$$h_f = f + h_{v_1} - h_{v_2} - m\ (h_{v_1} - h_{v_2})$$

or

$$h_f = f + (h_{v_1} - h_{v_2})\ (1 - m)$$

If $m = 0.5$

$$h_f = f + (h_{v_1} - h_{v_2})\ (0.5)$$

However, for a contracting reach, $h_{v_1} < h_{v_2}$, and the quantity in parenthes
is negative. It is made positive in the following way:

$$h_t = -\ m\ (h_{v_1} - h_{v_2})$$

Then

$$h_f = f + h_{v_1} - h_{v_2} + m\ (h_{v_1} - h_{v_2})$$

or

$$h_f = f + (h_{v_1} - h_{v_2})\ (1 + m)$$

If $m = 0.1$

$$h_f = f + (h_{v_1} - h_{v_2})\ (1.1)$$

Expressing an approximation of α in terms of section properties, K and
is essential to a direct method of determining the discharge of a stream
the slope-area technique. Consider the two-section reach 1-2 in Fig. D1.

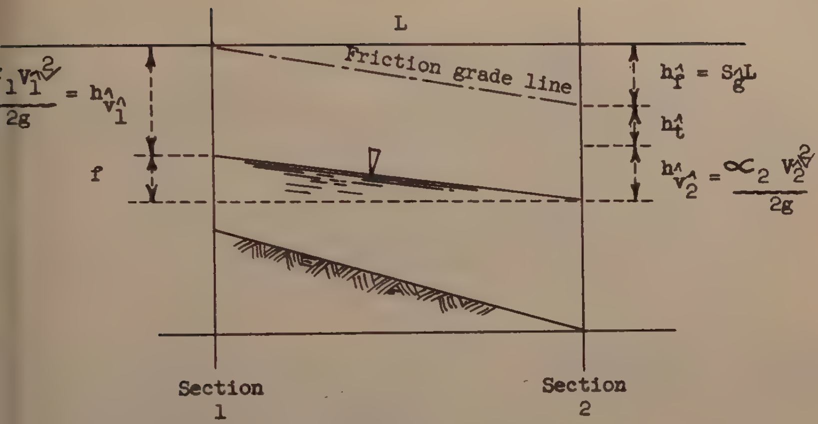

FIG. D1.

om continuity

$$Q_1 = K_1\, S_1^{\frac{1}{2}} = K_2\, S_2^{\frac{1}{2}} = Q_2$$

$$_1Q_2 = (K_1\ K_2)^{\frac{1}{2}}\, (S_1^{\frac{1}{2}}\ S_2^{\frac{1}{2}})^{\frac{1}{2}}$$

t is

$$Q = K_g\, S_g^{\frac{1}{2}}$$

o

$$h_f = f + h_{v_1} - h_{v_2} - h_t$$

r a contracting reach

$$h_t = -\, m\, (h_{v_1} - h_{v_2}) \qquad \text{and} \qquad h_f = f + (h_{v_1} - h_{v_2})\ (1 + m)$$

r an expanding reach

$$h_t = m\, (h_{v_1} - h_{v_2}) \qquad \text{and} \qquad h_f = f + (h_{v_1} - h_{v_2})\ (1 - m)$$

reover

$$h_f = S_g\ L = \frac{Q^2\ L}{K_g^{\,2}} = \frac{Q^2\ L}{K_1\ K_2}$$

then

$$\frac{Q^2 L}{K_1 K_2} = f + (h_{v_1} - h_{v_2})(1 \pm m)$$

which by algebraic manipulation becomes, for a contracting reach,

$$Q = K_2 \sqrt{\frac{f}{\frac{K_2}{K_1} L + \frac{\left(\frac{K_2}{A_2}\right)^2}{2g} \left[\alpha_2 - \left(\frac{A_2}{A_1}\right)^2 \alpha_1\right] (1 + m)}}$$

and, for an expanding reach

$$Q = K_2 \sqrt{\frac{f}{\frac{K_2}{K_1} L + \frac{\left(\frac{K_2}{A_2}\right)^2}{2g} \left[\alpha_2 - \left(\frac{A_2}{A_1}\right)^2 \alpha_1\right] (1 - m)}}$$

Modification of the area relationship by α values determines whethe reach is hydraulically contracting or expanding; that is, whether the weigh velocity head is increasing or decreasing in the downstream direction. If quantity in brackets is positive, the reach is contracting; if negative, the re is expanding.

Using data from the author's example, in the formula for a contract reach, the discharge was computed as 10,700 cfs.

PERFORMANCE OF FLOOD PREVENTION WORKS DURING THE 1957 FLOODS[a]

Discussion by Fred W. Blaisdell

FRED W. BLAISDELL,[1] F. ASCE.—Mr. Moore has presented interesting rmation on the operation of the SCS flood prevention structures in Texas, ihoma and Arkansas during the floods of April, May and June, 1957. Acding to Mr. Moore, the performance of the structures, designed for a 50-torm and subjected to greater than 100-yr storms, ". . . exceeded expecons. . ." The paper is devoted primarily to the performance of the emery spillways. This is well because it is noted that ". . . the use of veged earth spillways for flood prevention structures . . . is a material diverce from the accepted principles employed by the engineering profession in design."

Regarding the principal spillway, Mr. Moore notes that it is used to disrge water ". . . at a rate designed to prevent flooding of the stream chanbelow." A section through a principal spillway is shown in Fig. 3. It cons of a drop inlet, a barrel through the dam and a cantilevered outlet. The that the performance of the principal spillways is not mentioned is prob due to the fact that they performed as designed. The writer has been coned with the hydraulics of the principal closed conduit spillways since , and naturally was extremely interested in seeing how they came through e severe tests. In September, 1958, he visited a number of the structures. visits confirmed the supposition that the structures did their intended job complete satisfaction.

The practice of omitting stilling basins and simply cantilevering the pipe ond the toe of the fill, as shown in Fig. 3, is a practice that is also "a mal divergence from accepted principles." It was the performance of these ilevered pipe outlets that the writer was particularly interested in evalu. Compared to the size of the pipe, the storage between the crests of the cipal spillways and the emergency spillways is large. At many of the ctures, this storage pool was partially drawn down and refilled several s as a result of successive rains and many of the principal spillways ran apacity for long periods of time. This gave maximum opportunity for the lopment of the scour holes anticipated to form at the pipe exit.

The performance of the cantilevered outlets was much better than the er had expected. Typical scour holes are shown in Figs. B1 and B2. B1 is the outlet at Honey Creek Site No. 8E. There is a 17-in. concrete through the fill and the cantilevered pipe is 18-in. corrugated metal. The

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October, 1959, by Charlie M. Moore.
Proj. Supervisor, Agricultural Research Service, U.S. Dept. of Agric., St. Anthony Hydr. Lab., Minneapolis, Minn.

FIG. B1.

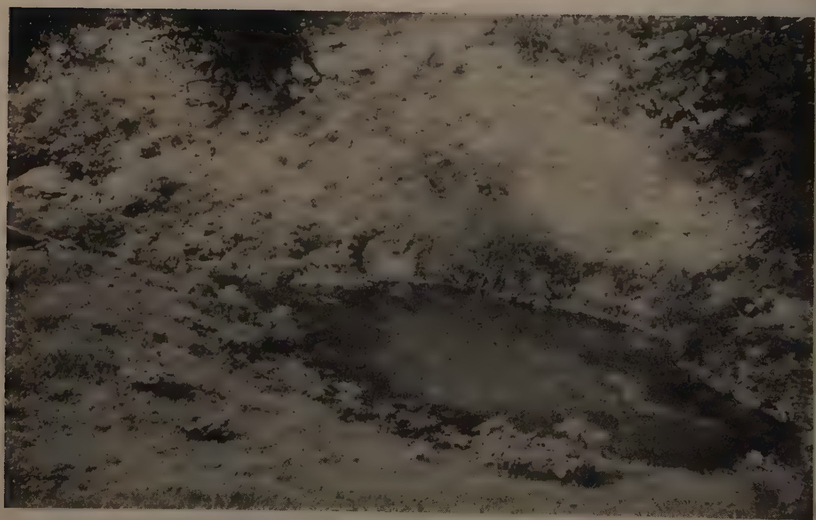

FIG. B2.

ran for six weeks at full capacity. The 12-in. outlet pipe shown in Fig. for the Honey Creek Site No. 14. The depth of the pool in the scour hole ut 4 ft. It can be seen in Figs. B1 and B2 that the scour holes are small e. At both sites, as well as all other sites seen, the scour hole termi- close to the end of the pipe. The bent shown in Fig. 3 is intended to sup- he pipe if the scour hole progresses upstream. The experience of the structures indicates that this feature may not be necessary under the ions existing there. However, it is not meant to imply that bents would iecessary in all cases.

e conclusion to be drawn from the Texas experiences is that a canti- ed outlet may be used in place of a stilling basin.

e writer has been looking at cantilevered outlets throughout the Midwest number of years with the expectation of finding some structures in which

FIG. B3.

our hole had reached a dangerous size. He recalls only perhaps two ures in which the scour holes looked as though they might eventually op to a damaging size but which had not yet progressed to that point. One se outlets is shown in Fig. B3. This is a 42-in. cantilevered outlet built 0. It carried a designed flow in 1952. This single flow formed the wide hole shown in the picture taken in 1954. Comparison with a picture in 1952, shows no visible increase in scour hole dimensions, possibly se additional capacity flows have not passed the structure. Both pic- show that the scour under the end of the pipe is insufficient to endanger ny way. In fact, the scour has not undermined the pipe as far as the bent led to support the pipe over the scour hole. In spite of the rather wide hole, the outlet performance must be classed as satisfactory. Whether

or not future capacity flows will increase the size of the scour hole to dan ous proportions is of course unknown at the present time.

The writer feels unsure of the performance of cantilevered outlets in of the fact that he has been unable to discover field evidence to back u doubts. He feels there must be limitations to the size of cantilevered ou that can be used with soils of different erodibilities in order to preven scour hole from endangering the dam. Here is a fertile field for resea Will the scour holes develop laterally, rather than in a dangerous upst direction? Will upstream development of the scour hole take place afte lateral development has reached a certain extent? How long will it take t velop the scour hole? What is the effect of the bed material? What will b ultimate size? Is this an economical self-formed stilling basin? Thes questions that need answering.

Evidence collected by the writer indicates that this type of outlet struc has a definite usefulness. Lack of difficulty with it is such that there i pressure from designers to conduct a thorough investigation into the cha teristics and limits of applicability of the cantilevered outlet. However writer feels that research on the cantilevered outlet would be of major v to those who design and use this economical outlet structure.

THE VORTEX CHAMBER AS AN AUTOMATIC FLOW-CONTROL DEVICE[a]

Discussion by Michael Amein

ICHAEL AMEIN,[1] M. ASCE.—The formation of vortices in flow, in hy-
ic structures, has been found undesirable because of three principal
res associated with flow containing a vortex. The first is the instability
head-discharge relationship, resulting in an erratic hydraulic behavior.
second feature is the reduction of the discharge capacity, resulting in in-
ed cost for the structure. The third feature is the possibility of damage
the erosive forces of the vortex motion. Such forces have been known to
damage to inlet drop spillway structures at earth dams. The authors
described a case in which the presence of a vortex is not accompanied
y of the undesirable features mentioned. They have experimentally
nstrated that the erratic behavior can be eliminated by stabilizing the
x in a vortex chamber. In the applications envisaged by them, the ero-
forces are either insignificant or irrelevant, and the reduction in ca-
would be used to advantage.

e type of vortex described by the authors usually results from flow
gh an orifice in the bottom of a tank, and is the closest approximation to
vortex. In a free or mathematical vortex, the product of the tangential
ity and the distance from the core is constant. This conclusion can be
ed by the application of Bernoulli's theorem along a streamline, and by
ing the radial pressure differential to the centrifugal force. In this
sis the effects of the viscous forces are neglected. However, since in a
luid vortex, viscous forces are present, the free vortex theory is not
ly applicable. Furthermore, from the principles of the free vortex mo-
s enunciated by Helmholz, the free vortex consists of the same fluid
les. In the real fluid, the fluid particles travel in continuously shrinking
around the core and are discharged through the orifice. Thus, the par-
constituting the vortex are constantly being renewed. It is the pre-
ance of the tangential velocity in the vortex chamber that gives this
a free configuration and behavior, so that the application of the theory
vortex in this case is probably correct for all practical purposes.

e authors have presented the results of their experiments in terms of a
cient of discharge and a dimensionless parameter called the vortex num-
This parameter contains the prime factors in the vortex motion. The
would favor the publication of the observed data in conjunction with the
s. A better understanding of the physical phenomena, the precision of

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ecember, 1959, by R.C. Kolf and P.B. Zielinski.
ydr. Engr., Anderson-Nicholas & Co., Boston, Mass.

measurements and the quality of experimentation could be obtained fro observed data. Although the presentation of data in dimensionless param is a commendable procedure following the widespread acceptance of d sional analysis, yet, this type of presentation, unless supplemented b served data, freezes the data to a single interpretation. Furthermor data would not be accessible to support or refute other approaches that come to light in the future.

The reduction in the discharge capacity of the orifice with the format the vortex can be said to be due to the conversion of part of the static h the tangential kinetic energy in the vortex. In the vortex chamber, the duction of flow at a tangential direction stimulates the formation of a v and tends to make the vortex a stable feature of the flow. The author' periments indicate that the coefficient of discharge C is a unique functi the vortex number, from which the stability of the head discharge relatic is deduced.

In the following analysis, three new symbols are used in addition t ones described by the authors: b = width of either of two inlet cha W = velocity in the inlet channel at the entrance to the chamber; and θ = between the inlet channel alignment and the tangent to the chamber at the of entry.

The velocity in the inlet channel is so small that the depression c water surface due to the velocity head is neglected. Then inflow

$$Q = 2\ H\ W\ b \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

The circulation is

$$\Gamma = \pi\ B\ W \cos\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

But, from the definition of vortex number

$$\Gamma = D\sqrt{2\ g\ H}\ \underline{\underline{V}} \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

Therefore,

$$W = \frac{D\sqrt{g\ H}\ \underline{\underline{V}}}{\pi\ B \cos\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

By substituting this value of W in Eq. 1,

$$Q = \frac{2\ b\ D\sqrt{2\ g}\ H^{3/2}\ \underline{\underline{V}}}{\pi\ B \cos\theta} \quad \ldots\ldots\ldots\ldots$$

The outflow is expressed as

$$Q = C\ A\sqrt{2\ g\ H} \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

From Eqs. 5 and 6, it follows that

$$H = \frac{\pi\ B\ A \cos\theta}{2\ b\ D}\ \frac{C}{\underline{\underline{V}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

indicates that the head H in a given vortex chamber can determine the x number and the discharge. For the case in which b = 6 in., B = 42 in., in. and $\theta = 20°$.

$$H = 2.1 \frac{C}{\underline{\underline{V}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

en the values of $\underline{\underline{V}} = 0.8$ and $\underline{\underline{V}} = 2.0$, the relation between C and $\underline{\underline{V}}$, as nined from Fig. 2 is

$$\underline{\underline{V}} = 3.1 - 4.6\,C \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

, by substitution in Eq. 8, results in

$$C = \frac{2.1\,H}{1.4 + 3.1\,H} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

s known, then C and Q can be readily calculated. Such a calculation is nted in Table 1.

is interesting to compare the head required to provide a given rge when there is vorticity in ow to the static head required to le the same discharge when the ity is eliminated. For the latter a constant C equal to 0.686 is ed. The static head required to le a discharge Q can be deter- from

$$H = \frac{Q^2}{2\,C^2\,A^2\,g}$$

TABLE 1.—COMPUTATION OF DISCHARGE

H, in feet	C	Q, in cubic feet per second
0.8	0.43	0.15
1.0	0.47	0.18
1.2	0.49	0.21
1.4	0.51	0.24
1.6	0.53	0.26

esults of the computations are shown in Table 2.

s seen from Table 2, that as the head on the orifice increases, a small- tion of it is converted to vortex motion. Therefore, the vortex chamber

TABLE 2

Q, cubic r second	H with vortex, in feet	H with no vortex, in feet	E, in %[a]
.15	0.8	0.4	47
.18	1.0	0.6	39
.21	1.2	0.8	32
.24	1.4	1.1	24
.26	1.6	1.2	22

represents the percentage of the total static head lost in vorticity.

may not be useful as a flow control device, that would maintain a consta
a slowly rising discharge under a rising head. In fact, as has been ex
enced in drop-inlet spillway structures, a sufficiently high head will d
out the vortex motion and re-establish the normal orifice flow relations
Many cases could exist in which the vortex chamber would perform no b
than an orifice of smaller area. It appears that it is the range of head
discharges within which flow takes place that determines whether the p
ence of a vortex in the flow can be permitted and whther the vortex ca
made to perform a useful function. Further investigation might indicat
even in hydraulic structures it may be more feasible, in some case
stabilize the vortex than to make attempts towards its elimination. Rese
on vortex motion would not only open new fields for its application, but
also add to our basic understanding of the mechanics of flow.

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PROCEEDINGS PAPERS

Th technical papers published in the past year are identified by number below. Technical-division nsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air nsport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hy-ulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil chanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors V). divisions. Papers sponsored by the Department of Conditions of Practice are identified by the sym-s (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning h Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To ate papers in the Journals, the symbols after the paper number are followed by a numeral designating issue of a particular Journal in which the paper appeared. For example, Paper 2270 is identified as 0(ST9) which indicates that the paper is contained in the ninth issue of the Journal of the Structural ision during 1959.

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